Special Relativity: The Central Ideas

Simple, Clear, Non-Mathematical
With Twelve Paradoxes

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SPECIAL RELATIVITY

Chapter 1:   Setting the Stage

Aim of Book

The aim is to explain, with almost no mathematics, the main facts of special relativity --perhaps the most intriguing subject known to the scientific world.

Focus is on the central phenomena that have puzzled experts for decades and caused beginners to tremble. Does the high speed of a rocketship whizzing past me really make the ship contract?   -Make its clocks run slow? -Increase its mass? -Slow down the aging of the crew? Many textbooks say Yes, a few say No, some hedge. Here we give the correct answers, briefly and convincingly.

The task is simplified by the use of short self-explanatory terms, easily-remembered rules (mnemonic aids), and illustrative cases (scenarios).

Incidentally, some widely circulated misinterpretations are examined, unmasked, and buried.

Paradoxes: The twin paradox is stated, proved in several ways, then explained. It is also exploited to drive home some seldom-appreciated basic facts.

Many other paradoxes are presented, including some brand new, highly challenging ones. Included are:

> The two separating spheres--yet there is only one
> Oppositely speeding travelers
> Four long trains speeding along four crossing tracks
> Long pole included in short barn
> Moving sidewalks that underperform
> Elliptical wheels
> Reversing helix
> Bursting wheel

The book is intended for all college students interested in science--and especially for physics students, physics teachers. Emphasis is on verbal explanations and on true, lasting, understanding.

Supplementary Reading May Be in Order

Persons seeking a more detailed and mathematical account of special relativity may wish to consult various available textbooks. Especially to be recommended is "Spacetime Physics" by Edwin F. Taylor and John A. Wheeler, 2nd edition, 1992; Freeman & Co. ISBN 0--7167-2327-1.

What Is Special Relativity?

It is the science of the relationships between physical objects (such as rocketships, persons, clocks, metersticks, electrons, protons, stars) that are traveling at constant high speed relative to one another. In particular, it is the science that shows how time and space are linked --truly linked in an exact and fully predictable manner.

The combination of space and time is called "spacetime". Time is called the fourth dimension.

In special relativity, the governing quantity, or crucial parameter, is relative speed, especially if it is very high, such as 10% or 50% or 87% of c, the speed of light. Such speeds are called "relativistic".

If the relative speed is less than 1% of c, the time-and-space linkage is of little or no consequence and there are no interesting phenomena. Forget about special relativity!

If the relative speed is extremely high, for example in the range between 90% and 100% of c, the linkage is highly important. Various phenomena seem amazing and incredible--until explained in terms of special relativity. This is abundantly clear to high-energy physicists working with beams of fast electrons, protons, pions, etc., and clear also to astrophysicists studying distant galaxies receding from our Milky Way galaxy at speeds comparable to the speed of light.

No acceleration, please! High-speed is central to the subject, but acceleration and gravitational effects are not. They are excluded.

Usually, especially in "thought experiments", we deal with systems that are freely coasting (non-accelerating systems, "inertial systems"). Often we assume that the systems are far away from any very massive objects and therefore are in zero-gravity regions --or, if there is gravity, the systems are freely falling ("freely floating") and so "do not feel the gravity." However, we often deal

with systems right here on earth--systems in which gravitational effects are negligible (example: beam of high-speed electrons in an "atom-smasher").

If the systems are accelerating, or are in very-high-gravity regions, major complications arise and a whole new science takes over, namely General Relativity --a difficult subject outside the scope of this book.

Basic Importance

Special relativity is of unsurpassed practical importance and also unsurpassed philosophical importance. Its practical importance consists of the fact that it is a "science of sciences." Together with mathematics and logic, it constitutes a foundation on which the physical sciences in general rest. Any law of any science that fails to conform to the laws of special relativity is automatically wrong. The laws of quantum theory fully conform to special relativity as do the laws of electromagnetic theory, thermodynamics, interstellar travel, and nuclear reactors.

The unique philosophical importance lies in the fact that special relativity gives man his first clear view beyond the limits that have been accepted since time immemorial, beyond the standard three dimensions (such as east-west, north-south, up-down), beyond the notion that time is totally distinct from space. Einstein's revolutionary discovery continues today to stimulate thinking beyond what formerly seemed to be an ultimate horizon. His discovery has even opened a Pandora's box of recent proposals (by experts on quarks and other fundamental particles) that our universe may in fact have more than four dimensions. Perhaps ten. Perhaps far more than ten. We have entered an era where people can contemplate kinds of space more complex than could be dreamed of a few decades ago.

The Two Branches of Special Relativity

Special relativity as a whole has two branches:

(1) Spacetime Geometry of Special Relativity

This is the science of comparing place and time data indicated by tools in one frame with the place and time data indicated by tools in another frame. It is a fascinating science, but "bloodless", having little or no concern for real properties of real objects. It is a

mathematical science--relying mainly on high-school math. It is a logical science. It is rich in intriguing philosophical implications.

(2) Applications of Special Relativity

This is the science of how relative speed affects physical objects, or at least affects measurements made on them--affects the objects' apparent properties. Momentum depends on relative speed. The same applies to kinetic energy, total energy, and various other physical properties. A beam of light that strikes an object imparts momentum and energy to it, and the amounts imparted are governed by the rules of special relativity. Kinetic energy and radiant energy can be converted into matter, and vice versa; and the processes conform to the rules of special relativity.

Persons working in the fields of high-energy physics (involving particles traveling at speed close to that of light, and involving energy-to-matter and matter-to-energy conversion) and in astrophysics (involving pulsars, black holes, etc.) are highly dependent on the rules of special relativity. The rules are of central importance in their daily work.

In this book, we deal with these two branches sequentially, starting with spacetime geometry.
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Before launching into the main facts and laws of special relativity, I must introduce the underlying concepts and key terms. These are explained below.

Introduction of Shorter Clearer Terms

Strangely, most authors of books on special relativity have timidly stuck with ordinary words --the standard words of pre-1900 physics. Yet they are faced with explaining phenomena entirely outside the scope of 1900 thinking. The result is that some discussions become wordy and foggy--rich in circumlocutions and misleading metaphors.

Why have those authors failed to introduce a set of truly suitable new terms? Because they themselves have learned to limp along with the old terms? Or because they believe that no one will buy a book that introduces a lot of new terms? Or because they have failed to take the time to work out new terms?

I suspect that this last reason is the real one. To work out ideal terms requires mulling over the actual phenomena and finding the true essence of each kind of behavior. To develop a set of terms that is self-consistent, brief, and self-explanatory is no easy task.

In this book I have introduced a number of new terms --to great advantage. Concepts are stated more clearly and briefly, relationships between them are clearer, and the key rules are more easily remembered.

Of course, the mathematical heart of special relativity is entirely correct and is presented well in standard textbooks. My effort has been concentrated on improving the verbal interpretation and assisting full understanding.

Acknowledgment:

I owe a great debt to my late colleague John C. Gray (1908 - 1995) for forcing me to re-think various central concepts, reject several conventional verbal explanations that are somewhere between vague and wrong, and bring in concise new terms.

I have benefited greatly from studying the Taylor-Wheeler book "Spacetime Physics" (1966 and 1992 editions) and from long and helfpul discussions with Prof. Edwin F. Taylor.

"Stationary" and "Moving" Are Always Relative

In the universe there is nothing that is "guaranteed stationary". Every galaxy is moving relative to every other galaxy, and in our Milky Way galaxy (or any galaxy) each star is moving relative to the others. The planets move with respect to the sun. There is no criterion for deciding what, if anything, is stationary. The same applies to the terms "moving" and "high-speed." Any freely coasting object has equal claim to be "stationary" or "moving" or traveling at "high speed".

A more recent advance in understanding was the history-making demonstration, by Michelson and Morley in 1887, that no "ether" could be found (and so cannot be used as embodiment of "stationary"). Earlier there was a common assumption that--even if no tangible object could

be said to be absolutely stationary--there was an all-pervasive "ether" and speed could be defined and measured with respect to it. Physicists assumed, in particular, that light had a constant speed with respect to the "ether". The Michelson-Morley experiments tossed this assumption into the trash can. The speed of light is not related to any "ether".

Equally shocking was Einstein's pronouncement in 1905 that the measured speed of light in vacuum is independent of relative motion of source and detector. It is independent of everything!

Conclusion: There is nothing that has any unique claim to any particular speed or direction of motion.

Happily, all that ever matters is relative motion, relative speed--and, of course, relative position. Again and again in situations involving time and space, the word "relative" looms large.

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Unfortunately, most authors occasionally slip up. Even after explaining at length why the bare words "stationary" and "moving" have no meaning, they proceed to use such words. They say, for example: "If you are stationary and the spaceship is moving, you find the following peculiar effects....." The unwary reader is likely to succumb to the notion that peculiar effects are caused by "motion" and are avoided if you remain "motionless."

Of course, scenarios such as; "Smith is stationary and Jones speeds past him." are very brief, clear, and graphic. But they stack the cards against understanding the underlying physics, the true causes and true effects. An analogy: The scenario "The sun rises in the east." is brief, clear, and graphic --but is bad physics (because it directs your thinking away from the true action: rotation of the earth).

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Key Quantities

Discussions of special relativity are centered on light, matter, position (place, distance and length), time, and relative speed. Also a key role is played by highly localized physical happenings (events) that serve as benchmarks.

Light Light is far more remarkable and mysterious than a layman can appreciate. It consists of the transfer of energy at incredibly high speed and 100% efficiency. The transfer may be over an enormous range of distances, distances smaller than a trillionth of an inch or greater than a billion light-years. When traveling in a vacuum for seconds, days, or millions of years, light experiences no friction, no degradation; 100% of the starting energy continues onward, and the polarization properties (wave forms) remain unchanged. According to the usual definition of speed, light travels at almost exactly 300,000 km, or 187,000 miles, per second. (A special, seldom-used definition, discussed in Chapter 23, has different implications.)

Maxwell, in 1873, discovered that light is connected with electrical and magnetic effects and indeed consists of waves of such effects. The same applies to radio transmissions and x-ray beams and also ultraviolet and infrared radiations. All of these, together with light, comprise a single family known as electromagnetic radiation. The family members differ in wavelength; waves of x-ray beams are less than a billionth of an inch from crest to crest, and at the other extreme the waves of long-wave radio broadcasts may be several miles from crest to crest.

What are the waves of? What is waving? Electric voltage, magnetic field strength. Is the wave motion forward and backward (like sound waves in air), or is it sidewise -- laterally? The answer is laterally--as in a taut rope.
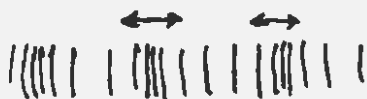
Backward-forward waves
(Longitudinal waves)

Sidewise waves
(Transverse waves)

A pulse of light travels as waves, but its production and demise --its start and finish, or birth and death, its actions -- occur in a quantized (packaged or particle-like) manner, each package being called a photon. Thus light has the conflicting properties of traveling like waves but acting like particles.

Light never stands still relative to any container or person, never can be weighed, has no "rest mass." Yet it has momentum and can add to the mass of a system that contains matter.

The speed of light has a mysterious "frame independence", of crucial importance. This is discussed in a later section.

Matter   Matter, which consists mainly of aggregates of protons, neutrons, and electrons, is another form of energy, a form that can stand still in a laboratory, can be weighed, and has rest-mass.

Matter may consist of individual particles, or large solid objects (stones, people, spaceships, planets, etc.),or quantities of gas, liquid, or solid.

Place, Distance, Length   Places are measured, of course, relative to suitable reference marks, or coordinate systems. If two places are measured, then, by subtraction, one finds the distance between them--the length of the straight line between them. .

Place (likewise distance and length) may be expressed in terms of feet, meters, kilometers, light-years, etc.

Note: I sometimes use "space" and "place" interchangeably. Both refer to our familiar 3-D surroundings. ("Spacetime" always has 4-D connotation.)

Time   This is mysterious and puzzling. You cannot grasp it, make a mark on it, label it, or weigh it.

Does it really exist? Or is it merely an idea in the brain of man? For practical purposes, it really exists. At any rate, there is a commonly accepted definition that permits precise measurement. It is defined as the number of cycles completed by some steadily cycling object, for example, the number of rotations of the earth (number of days), or number of swings of a pendulum (number of seconds), or number of vibrations of a molecule. Thus it is a distinct and objective quantity. (Quantity, yes; but what is its quality? What is it made of? Does time move along past us? Or do we move along through it? These questions have no good answers. But physicists do not care; they are happy with the quantitative definition, that is, with the numbers.)

Time may be expressed in nanoseconds (1 ns = 10^{-9} sec.), seconds, minutes, hours, days, or years.

<u>Terms "Cotimey", "Distimey", "Coplacey", "Displacey"</u>
In this book we use "cotimey" to mean "at the same time",
that is, "simultaneous", and we use "distimey" to mean "not
cotimey". Likewise we use "coplacey" to mean "at the same
place" and use "displacey" to mean "not coplacey". Use of
these terms makes discussions simpler and symmetric.

Preferred Units of Place and Time

For simplicity, we employ in this book the units <u>foot</u>
and <u>nanosecond</u>. The simplicity consists of the (fortuitous)
convenient fact that light travels almost exactly 1 foot in
one nanosecond. We usually use the simplifying convention
that the relationship is exact: 1 ft. of travel by light
takes 1 ns. (A more exact relationship is : 1 ft. of travel in
vacuum takes 0.984 ns.) When, in later chapters, we deal
with place and time as related dimensions, our
discussions are vastly simplified by using matched units.
(Most writers fail to use simplified matched units .)

Definition of Speed

Usually we express speed, v, in terms of feet per
nanosecond (ft/ns). Thus the unit of speed is 1 ft/ns.
Sometimes we use the unit 1 meter/second, or 1 light-
year per year.

On some occasions we define speed as the ratio v/c,
that is, as a fraction of the speed of light. Often this
simplifies thinking and simplifies discussions. The ratio
ranges, obviously, from 0 to 1. The ratio is often called
beta (β).

Use of "Event"

Essential to discussions involving two or more frames
in relative motion are universally observable physical 4-D
reference points-- highly localized happenings in place and
time. Anchored to such physical reference points,
discussions can be objective, down-to-earth, clear to all
observers irrespective of their relative motions.

We call such a reference point an "event".

Warning: As commonly used, an "event" can be small or large, short or prolonged. World War II was an event -- covering millions of square miles and several years. In our discussions we deal with events that are extremely localized, extremely brief--ideally, 4-D infinitessimal.

Ideally, in dealing with events, we regard them as frameless: each event exists in its own right and has no allegiance to any frame. Consider the collision of two atoms in interstellar space; the atoms collide and emit a flash of light, detectable from any and all frames. The event itself is frameless (or, if we can somehow associate a frame with it, this association is of no interest).

Other examples of localized and brief events: a spark, a small explosion, a lightning strike, a light pulse actuating a photoelectric detector, the hand of a clock passing the noon mark, a traveler arriving at a star. Every such event constitutes a 4-D reference point, detectable from all frames.

Is a fencepost at the end of my driveway an event? No--because no point in time is indicated. An event must indicate place and time.

Marks

We use "mark" to mean some physical indication in ordinary 3-D space only or in time only. A foot ruler has a mark every inch. A superhighway may have marks every mile. A typical clock dial has a mark for each minute. Special clocks may have marks for each second, or millisecond. Clocks employing cathode ray oscilloscopes may have marks for each nanosecond.

Notice that place marks are not concerned with time, and time marks are not concerned with place. (Events have 4-D significance.) Marks endure, events do not.

Two Kinds of Problems

In many problems in special relativity, we deal with interframe comparisons of time and space intervals between events. In other problem we deal with space (or time) intervals between marks. The two kinds of problems require different thinking, different procedures. (No standard textbooks mention this.)

Use of X-Axis and ΔX

In dealing with ordinary 3-D space, we usually employ Cartesian coordinates; we use x, y, and z axes that are mutually perpendicular and are marked off in similar units, for example, foot.

In dealing with two frames that are in relative motion, we usually assume that the line of relative motion is along the x-axis. Likewise we assume that the various objects of interest, and events of interest, are located along this axis, or at least are extremely close to it. Accordingly the y and z coordinates are zero or close to it--and therefore we do not need to mention them. This simplification is justified inasmuch as we can always, in principle, choose the location and orientation of the coordinate system in any way we please: we suit the coordinate system to the actual motions, events, etc.

Two space ships traveling
close to the x-axis

Adopting this simplification, we make much use of the symbol Δx, which represents the difference between the places of the two events as determined by a given observer-team--that is, the effective distance between the events (effective with respect to this team only). Sometimes we use the related symbols Δy and Δz.

Note: Usually we take Δt, Δx, etc. to be positive.

Because such symbols contains the Greek symbol "Δ", known as delta, we often call Δt, Δx, etc., "deltas."

Frame

The expression "frame of reference" or "frame" means the combination of an ordinary 3-D set of space coordinates plus some clocks that are in fixed positions in this set and are synchronized. The combination (4-D set of coordinates) has a fixed orientation (it is not rotating) and is coasting freely (it is not accelerating).

It may consist of a physical set of yardsticks (or metersticks or tape measures) and synchronized clocks, or merely a set that exists in our minds. The yardsticks indicate positions or distances along three axes, such as east-west, north-south, and up-down axes. The set is coasting freely in a region where there is no appreciable gravitational force--or where the pertinent region of space is so small and the relative speed is so high that the effects of gravity are negligible.

The frame is supposed to be of whatever size will comfortably accommodate whatever group of people or instruments is under discussion. The people, instruments, etc., are fixed in the frame. A group of persons or instruments that has a different direction and/or a different speed is said to be in a different frame.

Often, people or other objects are said to be in the same frame even if they are far apart. As an extreme case, persons on earth and persons on the nearest star may be said to be in the same frame --if earth's speed relative to star can be considered negligible.

(Persons who are accelerating --changing direction and/or speed --are in no one frame. Strictly speaking they are outside the scope of special relativity.)

Rest Frame (Proper Frame): This pertains to a specified object and is defined as the frame in which the object is at rest.

Nulling Frame: This is a frame with respect to which the distance or time-interval between two given events is null -- zero.

Symmetry (Neutrality) of the Two Frames Involved

A unique feature of this book is that, in dealing with two frames that are moving relative to each other, we consider the frames as having equal status. Neither is called "stationary", neither is called "moving". (Most writers use those terms--and thereby create prejudice. Readers may believe that measurements made by the "stationary" frame's instruments deserve greater weight than those made by the "moving" frame's instruments.)

To insure that our frames have equal status, I usually exemplify them by two spaceships that are at high speed relative to one another. No vestige of "stationary" applies to either one. Necessarily, attention is focused on <u>relative</u> motion. It is relative motion that produces the phenomena of special relativity.

Symmetry or Asymmetry of Deployment of Measurement Equipment

When we deal with two systems in relative motion, we must consider the symmetry or asymmetry of availability and use of the clocks, metersticks, etc. Are both systems equally equipped? Or does one system have much equipment and the other system none? Are all of the observations made from one frame? --All data obtained from one set of instruments?

Such issues are crucial. The outcomes of interframe comparisons depend on which frame's data we rely on.

Asymmetric Deployment of Equipment

Usually we assume that the location of measurement equipment is asymmetric -- one-sided. We assume that one and only one frame contains clocks, metersticks, etc., and includes a team of technicians (observers) for operating the equipment and recording the readings. Often we assume that we ourselves--or some mythical persons -- reside in the fully equipped frame and monitor the goings-on in both frames. (In some special cases it could be that "our frame" contains no equipment and the "other" frame contains much equipment.)

Of course, when persons in a laboratory make observations on a meteorite passing overhead, there are no actual clocks or metersticks in the meteorite frame, and accordingly we might conclude that the instrumentation is necessarily asymmetric. But the answer is not quite so simple --because the meteorite (1) may include molecules that are regularly cycling (rotating) and so can be regarded as very-high-speed clocks, and (2) may include crystals that have a definite atomic spacing and so can be regarded

as very-fine-scale measuring sticks. A stream of high-speed muons traveling across a laboratory room contains no clocks, but the muons themselves have a characteristic decay-time-interval (1/e lifetime--a small fraction of a second) that can be regarded as roughly equivalent to a clock.

Symmetric Deployment of Equipment

Sometimes both systems contain full sets of instruments: both include clocks, metersticks, etc. Thus there will be two sets of results, and the question arises: "How do the two sets of results differ? How are the differences to be reconciled?" Finding the answers is an exciting task!

(Most writers give little attention to situations in which the deployment of instruments is symmetric. Accordingly, they do not face up to the question: "How can the two sets of results be reconciled?" They miss out on some important insights.)

Universal Availability of Data

Often it is permissible to assume that the two fully-instrumented systems are so close together, briefly at least, that they can exchange data. They may be passing by each other at such short range that each can read the other's instruments, or the data can easily be transmitted from one system to the other by radio, TV, or FAX. In summary, it is entirely possible, in principle, for persons in each system, or indeed anyone anywhere, to receive all of the data from both systems.

(Most writers take it for granted, usually, that persons in one system remain totally oblivious to the findings of persons in the other system. Why? Perhaps to avoid confusion. But a possible consequence is that readers' (and writers') understandings remains incomplete.)

Is There an Observer "Standpoint"?

If only one system has measurement equipment, the resulting data and conclusions have a clear standpoint: this particular system, or frame.

But consider an investigator who receives data from

both systems' equipments. If the two sets of equipment are identical, various key results from the two sets may have equal magnitudes but opposite senses. The investigator may try to reconcile the conflicting implications -- may find reconciliation impossible, tortured (or fun!). But he has no standpoint. He is neutral.

In summary, if there is a standpoint, it is the location -- not of persons (analysts) -- but of the instruments used.

Naming the Frames

It is convenient to give frames names. Usually we assign to a given frame the name of a key person in that frame. Names that come readily to mind are the names used by the Armed Services to designate clearly (even in poorly transmitted radio messages) the letters of the alphabet, for example, Able, Baker, Charlie, representing A, B, C.

Thus we may say: "Ableframe is approaching Bakerframe at relative speed 0.87 c." or "Ableframe is coasting eastward past Bakerframe at 0.5 ft/ns."

Often we are more specific and deal with two spaceships that are in relative motion, and we regard the ships themselves as frames. We may say: "Ableship is approaching Bakership at 0.2 c."

Many of the more interesting problems dealt with in special relativity involve two frames that are in constant-high-speed relative motion. Usually the question is: "What do the instruments in one frame (Ableship, for example) show when they make measurements on objects and activities fixed in the other frame (the Bakership frame, say)?" Such interframe comparisons are the meat and drink of special relativity. The results are often surprising and puzzling --may seem self-contradictory -- until explained by the laws of special relativity.

Relative Speed Along a Line of Motion

A warning is needed. There is a kind of "speed" that is not relevant to special relativity. Consider two freely coasting spaceships that are in fast relative motion. Suppose their paths are one mile apart. Initially, when very far apart, they approach one another at high speed; that is, the distance between them decreases at a high rate. When

they are near one another, the distance changes slowly
(changes little or none when they are abreast), Soon
thereafter the distance increases at ever-higher   rate.
This kind of "speed" -- the varying rate at which the
distance between them changes--is not relevant to special
relativity.

What is important in special relativity is relative
speed along a line of motion. Consider two spaceships
Ableship and Bakership that are in relative motion relative
to one another. Consider a broad plane that (a) contains the
Bakership centerpoint and (b) is perpendicular to
Bakership's line-of-motion relative to Ableship. The speed
of Ableship relative to this broad plane is what we mean
by relative speed along a line of motion. This is the speed
we deal with in special relativity. For freely coasting
objects in empty space, relative speed, so defined, is
constant. It is constant whether the ships pass within 10
ft. of one another or 10,000 miles of one another.

Plane perpendicular
to line of motion
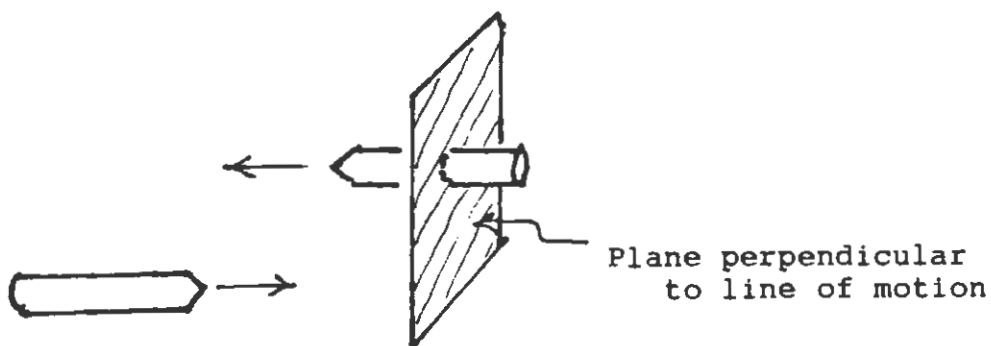
An equivalent way of defining relative speed is to
deal with the two pertinent frames, not the two physical
objects themselves, and assume that the frames are so
long and broad that they overlap and partly pass through
one another. The speed we deal with is the speed with
which they travel through one another.

Notice that the rules of special relativity apply as
well to, say, two spaceships that have a closest-approach
of 1000 miles as to ships that pass within one foot of
each other. The distance between a given ship and the
relative path (line of motion) of the other ship plays no
role. It can be entirely ignored.

Observers

Often, the key persons in a frame are called observers. It is assumed that such persons operate and monitor the various instruments in their frame. Of course, they do not rely on eyesight, since objects passing by at very high relative speed may be practically invisible, or just a blur. Rather, we assume that the observers rely on fast-acting highest-quality instruments.

Most writers assume that an observer monitors just the instruments in his own frame. But, as explained in a previous section, it is possible (in principle, at least) for an observer to learn the readings of instruments in other frames.

Life may seem simpler to an observer if he monitors only his own instruments and heeds only their readings. But he will obtain a more rounded, or balanced, understanding if he monitors also the instruments in other pertinent frames. Those observers who are expert on special relativity will be especially anxious to obtain a rounded understanding--and study all the data from all pertinent frames. (The author knows of no textbook that so much as mentions this point. Authors usually assume the observer to know nothing about special relativity and to have no interest in data obtained from another frame.)

Actually, the word "analyst" might be better than "observer". Precision instruments provide the data. The role of humans is to analyze and interpret it. Obviously, it does not matter what frame an analyst sits in --provided he has access to all the data he needs.

Tools

I use "tools" , or "instruments", to mean a given frame's metersticks, synced clocks, and other measuring instruments--such as those designed for measuring mass, momentum, photon absorption, electric field, magnetic field, particle disintegration, pair production. "Abletools" means the metersticks, synced clocks, etc. fixed in Ableframe and under the control of Capt. Able.

Broad Reality and Eifo-Reality

Here I introduce a topic that is crucial to the understanding of special relativity, yet is not mentioned in standard textbooks:

Concerning the reality of measured properties of physical objects, two distinct classes can be recognized:

> <u>property broadly recognized,</u> also broadly respected, from all frames. The value is extreme with respect to values found from all frames--it is the largest or smallest.
>
> <u>property narrowly recognized</u>--valid with respect to one particular frame only. The value is not an extreme. It receives little or no respect from any other frame.

I call these classes of reality "broad reality" and "eifo-reality". The term "eifo" is introduced formally in Chapter 8; it is the acronym for "<u>E</u>ffective with respect to the <u>i</u>ndicated <u>f</u>rame <u>o</u>nly." Note the underlined letters e, i, f, o.

Examples of broadly real properties are: the 100-yard length of a football field, the 24-hour period of rotation of the earth, the spherical shape of a child's balloon.

Examples of eifo-realities are: the football field's 50-yard eifo-length as measured by Ablecrew as Ableship coasts past the earth at 0.87c, the earth's 48-hour eifo-period of rotation as measured by that crew, the balloon's 2-to-1 oblate ellipsoidal eifo-shape as measured by that crew.

In many later chapters, broadly real properties and eifo-real properties are discussed and contrasted.

With the recognition of the two classes of reality, we can understand at once many topics that otherwise are obscure. In particular, most of the paradoxes of special relativity may be immediately understood.

It is amazing that textbook authors have failed to recognize the two classes, failed to name them --and have been content to resort to false or foggy explanations. Consider this sentence: "The football field is measured to be short." Or -with added fog--"The football field has, in some sense, its normal length, yet Smith's instruments show that it is greatly shortened."

How is that for ducking the issue? - Obfuscating it? How much simpler to say: "The football field's broadly real length is normal (100 yards), but the eifo-length found by Smith is short." Clear!

Scenarios

Many of the peculiar phenomena of special relativity are best understood by examining specific situations, illustrative cases, or scenarios. Some scenarios are entirely real, such as: "Consider a bunch of protons that are traveling along the Stanford linear accelerator at a speed (relative to the lab) of 0.999,999,999,5 c. Suppose we measure such-and-such by means of ......" But most scenarios are fictitious-- are mere thought-experiments, such as "If Able leaves earth and whizzes at 87% c toward star such-and-such....."

Frame-to-Frame Invariance of Rules

The key rules of special relativity are, of course, frame-invariant. If such-and-such results concerning Bakership are found by the Ableship instruments, exactly the same results would be found by the Bakership instruments concerning Ableship -- assuming similar circumstances on the two ships. The two frames have equal status, and therefore the laws of physics apply to them identically.

"Invariance" is a word that looms large in special relativity.

Place-vs-Time Symmetry of Rules

Concerning place and time there is a kind of symmetry-- more exactly, antisymmetry. This is discussed in a later chapter.

Note, however, that space and time have very different character. A person can be stationary in space, or can go back and forth. But a person cannot be stationary in time, cannot go back and forth in time.

No Cause-and-Effect Is Involved in Spacetime Geometry

Always one should bear in mind that measurements made in a frame A on objects in a frame B produce no effects on those objects. No cause impinges on them, and there is no effect. Strange-seeming results of measurements reflect on the measurement processes themselves, not on the object being measured. _

Each frame has its own special synchronism. Therefore measurements made from different frames necessarily yield different results, as explained in detail in later chapters.

It seems remarkable that many textbooks introduce, unnecessarily and wrongly, the idea of causality. They state explicitly, or imply, that high relative speed can "cause" shortening, or "cause" a clock to run slow. They imply that there is the process of shortening, the process of slowing, and the processes are "caused" by high relative speed.

Special relativity does not imply any cause or process. It is merely a set of rules that govern frame-vs-frame relationships.

For a 1/2-page summary of special relativity in words of one syllable, see Appendix 4.

For a knock 'em down, drag 'em out, account of the twin paradox, see Chapters 21, 22, 23.

For a set of tricky questions (and answers), see Appendix 3.

Chapter 2

The Six Basic Laws of Special Relativity

Introduction

Most of the remarkable facts of special relativity stem from six basic laws. A century ago, no reputable scientist would have accepted any of them. All seemed counter-intuitive, absurd, wrong. Today all are fully accepted.

In a sense, all of the mystery of special relativity resides in these laws. Once they are understood, the other strange and marvelous phenomena follow simply and logically.

Accordingly, these laws deserve the closest scrutiny, the hardest thought. Anyone who fails to understand them will remain forever unhappy with the other facts of special relativity.

Law #1: There is no preferred frame.

No one frame has unique properties, or optimum properties, or the simplest properties. All are on a par.

A century ago most scientists assumed that the sun, or the Milky Way as a whole, constituted a special frame-- a frame that could logically be called stationary. But this assumption was destroyed by the discovery in the 1920s and 1930s that there are countless other galaxies, all moving at different speeds with respect to one another.

Also, a century ago, many scientists believed that light traveled through some very real medium which they called "ether". They assumed that the ether constituted a frame, and they assumed that this frame constituted a definition, or embodiment, of "stationary". But when Michelson and Morley found that light beams in their labororatory had the same speed irrespective of whether the earth was moving toward a certain portion of the universe (in spring, say) or was moving away from that portion (in the fall), scientists soon came to the conclusion that there is no ether. (Or, if there is, it has no effect of any kind and so there is no point in ever mentioning it.) Thus died the idea of ether as the

embodiment of "stationary."

Corollary 1: The terms "stationary" and "moving" are meaningless. Likewise "slow", "fast". They have meaning only when we know "with respect to what?"

Corollary 2: The terms east, west, north, south, up, and down are meaningless--unless we know "with respect to what?"

Law #2: The speed of light is constant.

Always light traveling in vacuum has the speed, 3×10^8 m/sec., or about 1 ft./ns.

If you measure the speed of light ever so carefully, you will get the same answer whether you are moving toward or away from the light source or (which is the same thing!) whether the light source itself is moving toward or away from you. Even if you are moving at extremely high speed relative to the light source (or vice versa), your measurement of the speed of light is unchanged. Nothing that you can do, and nothing that can be done to the light source, can change the measured value of the speed of light in vacuum.

To scientists of 100 years ago who were familiar with sound waves in air, or sound waves in water, or waves on the surface of a pond, the stubborn 100% constancy of the speed of light seemed hard to believe. Only after the experimental proof had been refined, repeated, and double-checked did they accept this bizarre fact. Acceptance was facilitated by soon-to-follow pronouncement by Einstein that time and space are not the clean-cut things that everyone had assumed but are linked--joined in a certain definite manner.

Law #3: Nothing Can Travel Faster Than Light

No star, meteorite, bullet, proton, electron, quark, or neutrino can go faster than light. Nothing whatsoever -- not even x-rays or radio waves or gravity waves or energy of any kind -- can travel faster than light travels in vacuum, at $c = 3 \times 10^8$ m/sec. This is an absolute upper limit applicable to all branches of science.

Thus the speed c is much more than a property of light. It is a property of the universe as a whole. It constitutes a limit on every kind of process, whether in the field of physics, chemistry, communications, or astrophysics, or any other field. It would be enormously important even if eyes had never existed and light itself were unknown.

Warning: Outside the scope of Law #3 is the speed of a locus, such as the intersection point of the two blades of a pair of scissors as the scissors are being closed. Here, there's no upper limit on speed.

Law #4: No Matter Can Travel as Fast as Light

Nothing that has rest-mass (that is, no matter) can travel as fast as light. No star, meteorite, spaceship, baseball, dust particle, proton, electron, or quark can attain the speed of light.

Law #5: Events Found by One Frame's Instruments to Be Cotimey Are Found by Another Frame's Instruments to Be Distimey

Stated differently, the question "Are these two events cotimey?" has a frame-dependent answer.

Note: In this book we usually use "cotimey" in place of "simultaneous" -- because (1) it is briefer, (2) its meaning is more vivid, and (3) it invites use of a simple word for the opposite condition: "distimey". Most writers have no simple term for "distimey"; they use long expressions such as "fail to be simultaneous." This is wordy, tedious.

Proof of Law: The law is easily proven with the aid of a simple scenario, or thought experiment. Suppose the two spaceships Ableship and Bakership are passing by each other at high relative speed, with Ableship moving to the left toward Vega and Bakership moving to the right away from Vega. Suppose also that each ship has two short projecting antennas, or masts, and that the ships pass by one another so closely that the masts will touch. When one mast touches another, a spark is produced and a light pulse spreads out in all directions. (Assume, for simplicity, that only two sparks are produced--spark produced when Able's bow mast encounters Baker's stern mast and when Able's stern mast encounters Baker's bow mast.) Suppose, finally, that Able is seated midway between the Ableship masts, and Baker is seated midway between the Bakership masts--both seated externally so as to directly receive light from every spark. (See diagram drawn by Able.)
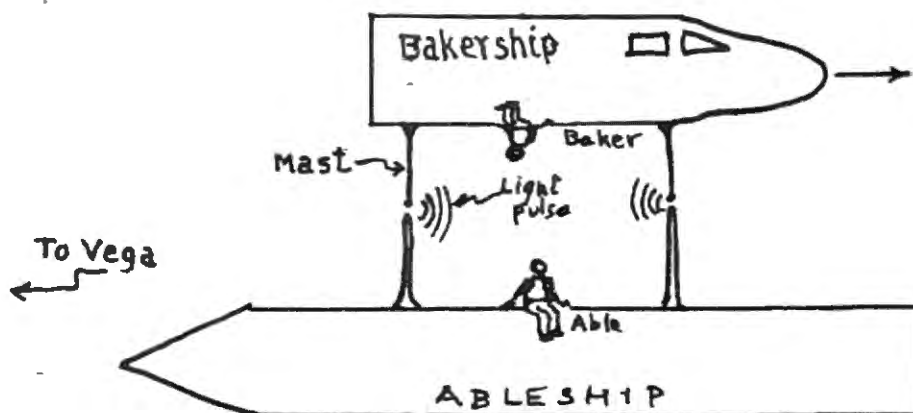

Diagram by Able

We now postulate that Able receives the light pulses (from the two sparks) at the same instant: each pulse has spread from its point of origin toward the two men, and it so happens that the two pulses reach Able at the same instant.

What does Able conclude? That the pulses originated at the same time--the two sparks were cotimey. This conclusion is inescapable because (1) he knows he is equidistant from his two masts and (2) the two pulses traveled at the same speed with respect to him (per Law #2). Same distance, same travel speed, same arrival time of pulses! Conclusion: the sparks occurred at the same instant--were cotimey.

What about Baker? Does he too conclude that the sparks were cotimey? No! While the pulses were traveling toward the two men, Baker was moving to the right, that is, away from Able and away from Vega. Therefore the pulse from the left spark reaches him after it has reached Able, and the pulse from the right spark reaches him before it has reached Able. In summary, Baker receives the right pulse before receiving the left pulse.
What about the distances, as judged by Baker? They are equal, because he is equidistant from his ship's masts. What about the speeds, as judged by Baker? These also are equal (per Law #2).
Baker's conclusion is clear: the right spark occurred first; the sparks were distimey. The conclusion is inescapable because of the equal distances, equal speeds of travel of light, and arrival of right-spark light earlier than left-spark light.

Can anyone doubt this? If so, suppose Able's brother is seated just to the right of Able. Then because Able receives the two pulses at the same instant, the brother will receive the pulse from the right source first. And it may be that, as he is receiving it, he is immediately adjacent to Baker--practically touching him. Therefore Baker also must be receiving the right pulse first.

Can a valid claim be made that, because the diagram is symmetric, the two men must arrive at the same

result? No. Our hypothesis that Able finds the two sparks occurred at the same time destroys the symmetry.   Note in particular that the diagram is as drawn by Able--has built-in bias. A diagram made by Baker would show a very different situation--highly asymmetric. See the following diagram,

<u>Warning concerning length effects</u> It turns out that, in this scenario, if the two Ableship masts are 100 ft. apart per the Abletools (metersticks, clocks, etc.), the two Bakership masts are necessarily more than 100 ft. apart per the Bakertools. The diagram presented above is as if drawn by Able. Baker would draw a very different sketch, something like the one shown below. (Length effects are discussed in Chapter 10.)
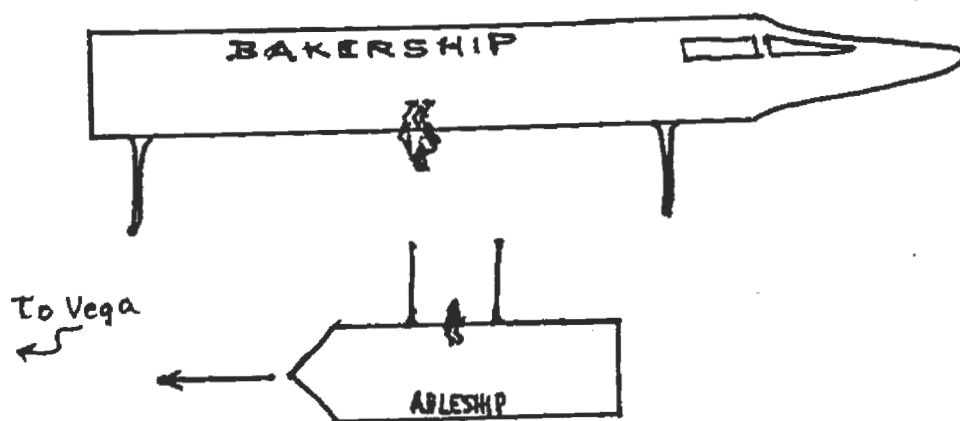

Diagram by Baker

<u>Short form of Law #5:</u> "Cotimey-to-Able implies "Distimey-to-Baker." And vice-versa.

<u>Note concerning symmetry:</u> If the sparks had been found cotimey by Baker, Able would have found them distimey.
    The above discussions are entirely neutral. There is no implication that either ship is "stationary" or is "moving faster" than the other. If either ship finds the sparks cotimey, the other ship finds them distimey.

<u>What would an "Intermediate Referee" find?</u> Suppose a referee were to travel so as to be equidistant from Able and Baker at all times. What diagram would show his findings as to the lengths etc. of the two ships as they pass one another? See the following diagram. (Note: Our various hypotheses embodied in Able's diagram necessarily mean that the proper length of Ableship differs from that of Bakership, and the proper distance between Ableship masts differs from that between Bakership masts.)
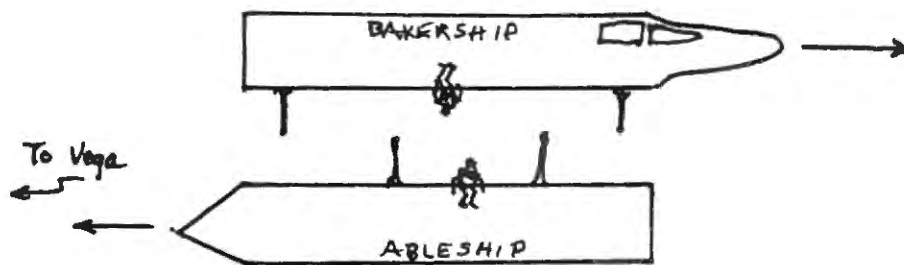

Diagram by intermediate referee

<u>Warning to anyone who has doubts:</u> The basic proof presented in previous paragraphs should be thoroughly understood, agreed to, accepted. Every step in the proof consists of simple physical facts. Any misgivings should be scrutinized and overcome. Otherwise the remainder of this book may seem unconvincing.

Yes, the result may seem very strange. The strangeness originates in Law 2: that the speed of light is the same judged from all frames.

Law #6: Events Found by One Frame's Instruments to Be at a Single Place Are Found by Another Frame's Instruments to Be at Different Places.

More briefly: Events found by Ablecrew to be coplacey are found by other frames' crews to be displacey. That is, the answer to "Are these two events coplacey?" is frame-dependent.

Proof: No proof is needed. Everyone who has driven a car knows that, from one second to the next, the windshield (for example) seems to be in the same place-- same relative to himself. But to a person lounging on the sidewalk the windshield is in very different places from one second to the next. An astronaut orbiting the earth has the impression that his spaceship is stationary, but to the Cape Canaveral tracking team he is in very different locations from one second to the next.

The Law can be compressed to: "Able coplacey, Baker displacey."

(If this law is obvious, why do we bother to state it? Answer: to bring out, and stress, the fact that time and place are sister dimensions. If there is a law "Cotimey-to-Able implies distimey-to-Baker", there must be a law "Coplacey-to-Able implies displacey-to-Baker." In the study of special relativity, it is often constructive to seek the parallelism between effects concerning time and effects concerning place. If one fails to find parallelism, one's understanding is incomplete.)

***

Absence of Transverse Effects

There are no transverse relativistic effects. More exactly, there are no effects pertinent to a set of events situated along a line transverse to (perpendicular to) the line of motion, and likewise there are no relativistic effects pertinent to a set of clocks or metersticks situated along a line transverse to the line of motion.

If two sparks that are cotimey per Able occur along a line exactly transverse to the line of Baker's relative motion, they are cotimey to Baker also. The interesting phenomena of special relativity pertain to events situated at different locations along the line of motion. Always what matters is the component along, or parallel to, the line of motion. All this is explained in Chapter 11.

Question as to Object-Orientation Effect

Relativistic effects always depend on the along-line-of-motion positions (of clocks or metersticks or other objects). If Bakership, on passing by Ableship, happens to be oriented transverse to the line of motion, that is, is coasting along "sidewise" relative to Ableship, no relativistic effects apply to Bakership objects located parallel to the Bakership axis. Effects apply only to those objects that lie along lines having components along the line of motion.

No To- Vs.-Fro Effects Concerning Length, Period, Mass

Whether an object is traveling toward you or away from you, your tools show the same relativistic effects -- same eifo-lengths, same eifo-periods (and eifo-rates) of clocks, same eifo-masses. What matters is the relative speed (a scalar). Whether the two given frames are approaching one another or are becoming farther and farther apart is immaterial to the values of the eifo-quantities. (Warning: this does not apply to an event-pair.)

List of Rules

Besides the six main laws of special relativity, there are several rules, or mnemonic aids that often prove useful. Among these are:

> Co=small rule . See Chap. 7.
> Co-change rule. See Chap. 5.
> Coplacey-distimey rule. See Chap. 2.
> Cotimey-displacey rule. See Chap. 6.
> Cotimeable-noncoplaceable rule. See Chap. 4.
> Coplaceable-noncotimeable rule. See Chap. 4.

Implied Corollary of the 2nd Law of Special Relativity: the Corollary "Clocks Perform Identically in All Frames"

How can anyone believe that the speed of light is the same measured from all frames unless he believes also that clocks of standard type perform identically in all frames? This latter belief is implicitly assumed in the "constant speed" law.

Notice that the "perform identically" corollary rules out the all-too-prevalent notion that "speeding clocks run slow".

***

We are now ready to explore the main phenomena of special relativity.

Chapter 3: Synchronization

Introduction

Special relativity has strange implications concerning time intervals and place intervals between two events -- intervals called Δt and Δx. (Note: events, not marks.)

Before Einstein upset various main laws of physics, the time interval between two events (two explosions, for example) was believed to be the same for everyone. If the time interval was one second according to your measurements, it would be one second according to everyone's measurements, no matter what their relative speeds --so it was believed. So deeply entrenched was this belief that no one even thought to put it into words. It seemed self-evident.

The same applied to place intervals. If two events (explosions, say) were 100 ft. apart according to your measurements, they would be 100 ft. apart according to everyone's measurements, so it was believed. This too seemed self-evident.

These beliefs crumbled into dust when Einstein expounded his newly discovered laws (see the previous chapter). Particularly pertinent was the discovery that events that are cotimey judged from one frame are distimey judged from all other frames.

Most of the strange new conclusions depend strongly on the synchronization (sync) of clocks in any given frame. The key fact is that each frame has its own special sync. Clocks in a Frame A that all read the same judged by observers in this frame are found to read differently as judged from all other frames--as explained below.

Just what is synchronism? How is it achieved?

The Synchronizaton Process

Suppose that Ableship contains two fixed clocks situated 10 ft. apart along the ship. To synchronize them --make them alike with respect to the Ableship frame-- Able, carrying a "master wristwatch", may walk from one clock to the other. As he passes each clock, he adjusts it to read the same as the wristwatch. If, after doing this, he walks back and forth from one clock to the other, he always finds agreement between the wristwatch and the clock he is passing by.

Warning: In moving along from one clock to the other, he must move slowly. Were he to move fast --say at 10% of c or faster--trouble would arise; he would find that when he walks at different speeds or reverses the direction of walking, the clocks appear out-of-sync. Therefore it behooves him to move slowly. The requirement is not stringent, because, in most instances, "slowly" covers every speed up to, say, 10,000 mph. No significant trouble arises unless he moves at relativistic speed, meaning a speed appreciable relative to c.

A more sophisticated method is to place a pulsed light-source midway between the two clocks to be synced. A short pulse of light is emitted (in all directions), and each clock, on receipt of the pulse, is set to (say) noon.

Various other procedures involving radio or light pulses are equally effective.

Such procedures can be used in syncing any number of clocks.

When all of the clocks in a given frame have been synced, they will all read alike according to any test made by persons that are stationary, or practically stationary, in that frame.

Synchronization Is Frame-Dependent

If there are several clocks situated along Ableship and these have been synced by Able, and if there are likewise several clocks situated along Bakership and these have been synced by Baker, each man judges the other man's clocks to be non-synced.

Proof: This follows from Law #5. Baker's clocks, having been accurately synced by Baker, all strike noon at the same instant, according to Baker. But according to Able, no two of Baker's clocks strike noon at the same instant. Why not? Because of Law #5, which states, in effect, "If Baker finds two events to be cotimey, Able finds them distimey."

The following sketch indicates the situation as judged by Able. The sketch applies to the situation when, judged by Able, Baker's bow clock reads noon. Notice that all of Able's clocks read alike, judged by Able (for example, they may all, by luck, read noon), but, again judged by Able, <u>no two of the Baker clocks read alike.</u>
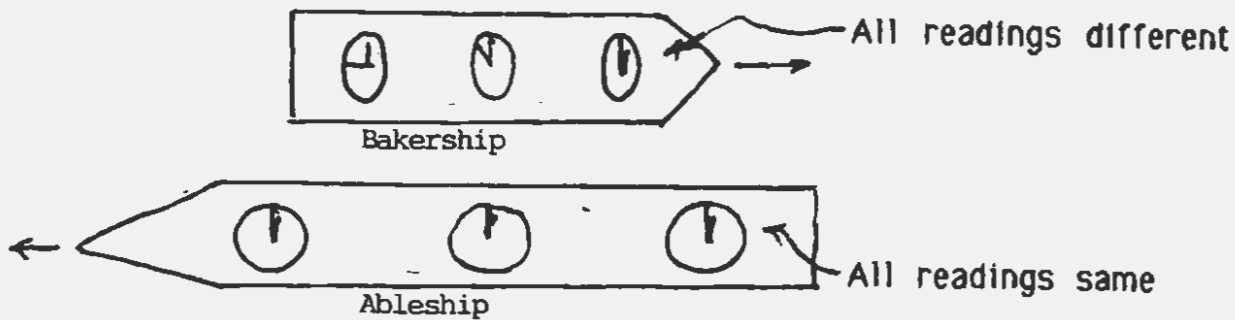

All readings different

Bakership

All readings same

Ableship

Diagram as drawn by Able

The conclusion can be compressed into the following rule: Able's sync is Baker's non-sync. Or merely: Sync is frame-dependent.

Notice that these findings follow directly from Law #5. Once we have accepted that law, the present finding follows directly. There is no additional mystery. The mystery stems from Law #5, which in turn owes its mystery to Law 2.

Warning: We assume that the clocks are arranged along a line parallel to the line of relative motion. If the clocks were situated along a transverse line, our conclusion would no longer hold. Ableship clocks that are situated along a transverse line and have been synced by Able are found by Baker also to be synced. (But if Charlie, in Charlieship, came along on some <u>oblique</u> course, he, of course, would find the clocks non-synced. Only if the alignment of clocks is strictly transverse to the observer-frame's line of motion is the sync frame-<u>independent.</u>)

Impossibility of Making an "All-Frames-Valid" Diagram

The facts presented above show that no one diagram can show both what the Abletools find and also what the Bakertools find. In the above-presented scenario the Abletools show the Bakership clocks to be non-synced, but the Bakertools show them to be in perfect sync. (Why so? Because what is one instant according to Able is not an instant -- has a considerable spread -- according to Baker. This is discussed below.)

For each observer-frame, a different diagram is needed-- a fact that makes life difficult for persons trying to depict special relativity scenarios.

"Same Instant" May Be Meaningless

The fact that sync is frame-dependent is highly important. For example, it is important to Able when he is trying to measure the length of Bakership which is passing by at high relative speed. Able wants to make measurements at two locations (locations of the two ends of the ship) at the "same instant." But what is "same instant"? A single instant defined by two clocks fixed and synced in Ableframe is --according to clocks fixed and synced in Bakership--two different instants.

Conclusion: When anyone mentions "single instant" he should say "with respect to what frame."

"When" May be Meaningless

In discussions involving several frames, a phrase such as "When Able coughed ..." has no meaning. "When" judged from what frame? The cough may have occurred before noon, according to Ableship clocks and after noon according to Bakership clocks.   A cough is distinct enough -- a clear-cut event. But what does "when" mean?

Always one should indicate: "When according to clocks in what frame."

Likewise one should not say: "While Smith was climbing up...."   or "During my breakfast..."   or "immediately following his speech..." or "at the moment of his arrival..." In each case, one should specify "judged from what frame?"

Front First Rule:

In the Law #5 scenario, Able's instruments show the two sparks to be cotimey and Baker's instruments show them to be distimey. Which spark occurred first, according to the Baker instruments?

Answer: The spark at Bakership's bow. The first light pulse to reach Baker is the pulse from the bow. Distances and speeds were same for both, according to him. Therefore he concludes that the bow spark occurred first.

Proof: The Law #5 scenario (see Chap. 2) shows that, if Able finds the two sparks to be cotimey, Baker finds the sparks distimey and in particular finds that the spark nearest his bow occurred first. More exactly: he finds that the spark farthest along in his direction of relative motion occurred first.

This conclusion can be compressed into: "Front event First" or "Front First". Or: "FF rule."

Warning: The FF rule does not apply to every pair of events --but only to a pair that, acording to some observer team, is cotimey. In general, If instruments in some one frame show two events to be cotimey, instruments in any other frame will show them to be distimey --with the front (farthest along) event occurring first.
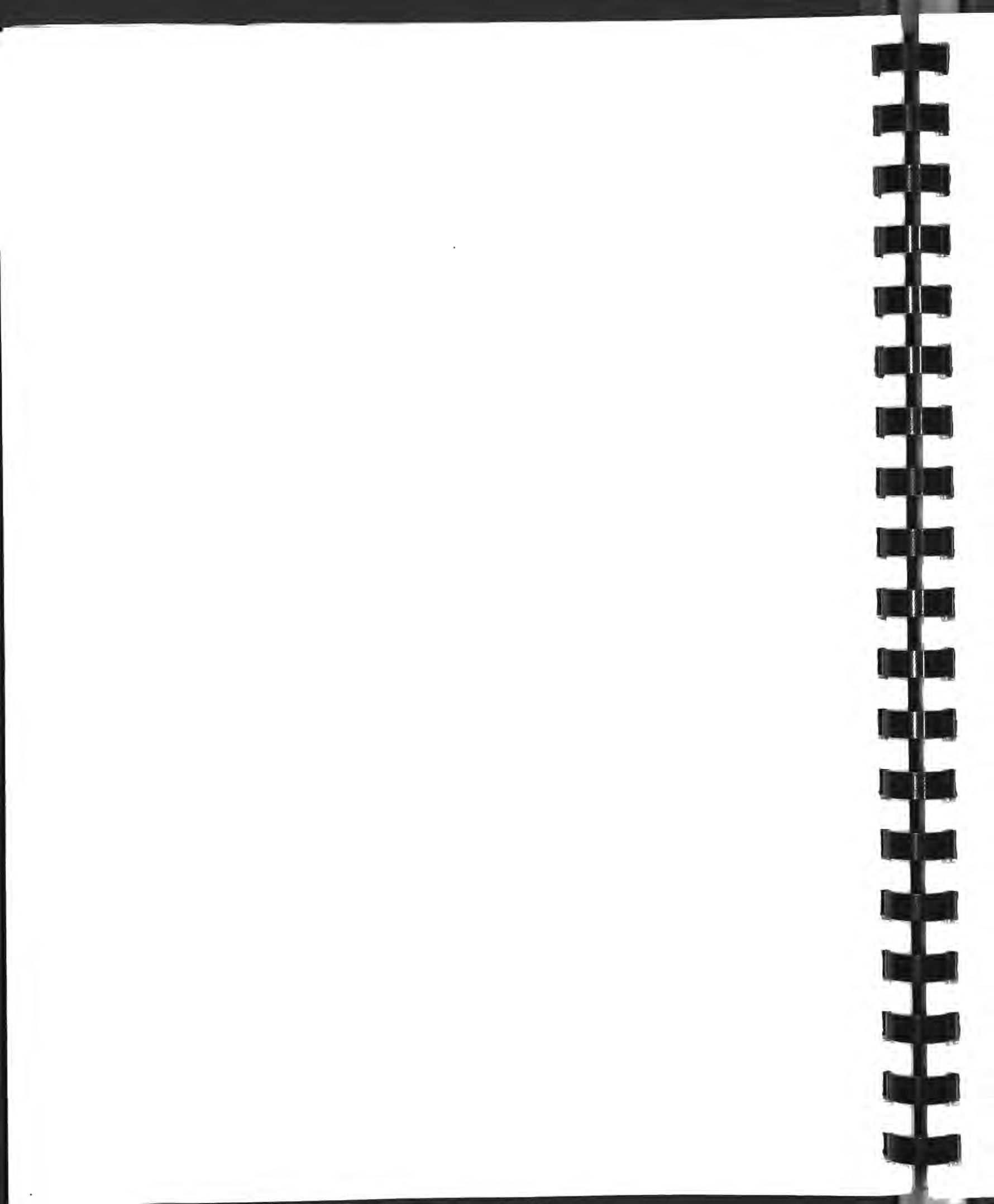
Chapter 4
Event-Pairs : the Three Classes

Introduction

Countless problems in special relativity involve a pair of events, such as two lightning strikes some distance apart and some time apart. Or two light pulses received at a given station. Or a departure from Boston and an arrival at New York. Or any other two events or happenings each of which is highly localized in time and space.

By dealing with pairs of events we keep the discussion physical and clear (rather than metaphysical and vague).

There are three classes of event-pairs, and they have different implications. One type is called "time-rich" , another is called "place-rich", and the third is called "non-rich".

(Warning: Most authors use the terms "time-like", "space-like" and "light-like". The suffix "like" seems ill-suited, not helpful.)

Time-Rich Event-Pair:

If observers in a given frame find that the time interval between two given events exceeds the place interval between them, the pair is called time-rich. Of course, matched units must be used. We use nanosecond (ns) and foot (ft). (Some authors use "time it takes light to go a meter" and "meter" --or "meters of time" and "meters." Awkward usages!)

Examples:
> You throw a stone, and later it breaks a window.
> I plug in my radio-set, and the loudspeaker starts producing music.
> My dog barks at noon EST, and at 2:00 pm EST the President coughs.

For each of these pairs of events, the time interval between events exceeds the place interval--assuming matched units are used.

Notice that the two events may be related (first two examples above) or totally unrelated (third example).

PLace-Rich Event-Pair:

If observers in a given frame find that the place interval between two given events exceeds the time interval between them, the pair is called place-rich. Again, use of matched units is assumed--ft. and ns, for example.

Examples:

My dog barks in Cambridge at noon EST; 20 ns after noon EST the President coughs in the Oval Office.

A pigeon takes off from Central Park at noon EST; 5 ns after noon EST a car hits a tree in Miami.

For each of these pairs of events, the place interval far exceeds the time interval.

Non-Rich Event-Pair:

If instruments in a given frame indicate that the time interval between two given events is exactly equal to the place interval between them (assuming use of matched units) the pair is called non-rich.

Example: A light source on earth is pulsed and the pulse is detected by a photocell on the moon. (Assume that the pulse travels straight toward the moon and travels in vacuum.) Then the time interval and place interval are identical)

Summary:
Time-rich: $\Delta t > \Delta x$.
Place-rich: $\Delta t < \Delta x$.
Non-rich: $\Delta t = \Delta x$.

Richness Type Is Independent of Frame

If a pair of events is time-rich as judged from one frame, it is time-rich judged from all frames.

Likewise if a pair of events is place-rich judged from one frame, it is place-rich from all frames.

A similar statement applies to a non-rich pair of events.

These considerations simplify many discussions.

Rule: Richness type is frame-invariant.

"Connectiveness" Possibility for Events of a Time-Rich Pair

Recognizing that, for every time-rich pair of events, $\Delta t > \Delta x$, we see that it is possible, in principle at least, for a person or clock or meteorite or electron, etc., to be present at both events. If a person travels at high enough speed (relative to the frame with respect to which Δt and Δx have the given values) he can be closely adjacent to the first event and also manage to be closely adjacent to the second. In other words, the events can be "connected" by a traveling person or other object.

(The reader will remember, of course, that we are dealing here events, not locations. It is always possible, for example, to travel from Boston to New York, but to travel from a noon explosion in Boston to a noon explosion in New York (same noon!) is impossible, because the two events --the two explosions-- are not a time-rich pair.)

Causality Possible with Time-Rich Pairs

The facts presented above prove that the two events of a time-rich pair can have a "cause and effect" relationship. The first event could be the firing of a gun and the second event could be the consequent breaking of a window. Or the first event could be the issuing of a radio command from an aircraft carrier and the second event could be a responding action by an airplane 500 miles away.

In summary, a time-rich event-pair can embrace causality.

Often, of course, the events of a time-rich pair have no causal connection. Consider a lighting strike in Boston at noon (per a Bostonian) and a child's balloon bursting in Singapore a week later. This is a time-rich pair, but no cause-and-effect is involved.

However, a very large number of time-rich event-pairs that are of interest to people do have a causal relationship. For this reason the subject of time-rich pairs deserves much attention.

Causality Not Possible With Place-Rich Event-Pairs

Between the events of a place-rich pair, no causality is involved. A causal connection is impossible--because no object or light pulse can be present at both events of such a pair. Too much distance, too little time.

Causality Possible With Non-Rich Event-Pair

A light pulse leaves Earth and travels directly toward the star Canopus. The departure and arrival constitute two events: a non-rich pair. With respect to any and all observers, $\Delta t = \Delta x$.

No person or electron or other bit of matter can be present at both   events of a non-rich pair.

But a light pulse can.

Thus the events of such a pair can be causally related --related by something traveling at the speed of light -- light itself or radio waves or x-rays, etc.

(Of course, if a light pulse travels in water or glass and so has a speed less than c, the departure and arrival of the pulse constitute an event-pair that is time-rich,)

Summary as to Possibility of Causality Link

The two events of a time-rich pair can be causally linked by traveling matter or a beam of light.

The two events of a non-rich pair can be causally linked by light or other signal having speed c--but by nothing else.

The two events of a place-rich pair cannot be causally linked,

Cotimeable-Noncoplaceable Rule

For any place-rich event-pair, one can find a frame with respect to which the events are cotimey (but no frame with respect to which they are coplacey).

Coplaceable-Noncotimeable Rule

For any time-rich event-pair, one can find a frame with respect to which the events are coplacey (but no frame with respect to which they are cotimey).

Richness of an Event-Pair

Let this be defined as $\sqrt{(\Delta t)^2 - (\Delta x)^2}$.
This is identically the Lorentz invariant interval (L-interval) discussed in Chapter 5,

Chapter 5
Lorentz Invariant Interval

Introduction

Here we deal with one of the most surprising and helpful concepts of special relativity: the spacetime interval between any two given events. It combines the time interval Δt and the space interval (combination of Δx, Δy, Δz). In all, it embraces four intervals --four dimensions. It is a "4D" interval.

Most writers call it the Lorentz invariant interval. I prefer to call it L-interval, which is briefer, easier to remember, easier to say and write.

Definition of L-Interval

If the two given events lie along the x-axis, and if some observer Able evaluates the space interval Δx between the events and the time interval Δt between them, his expression for the L-interval is very simple:

$$\text{L-interval} = \sqrt{\left|(\Delta t)^2 - (\Delta x)^2\right|}$$

(Note: The two short vertical lines in the expression mean: "Take the magnitude", or in other words, "If the quantity is negative, ignore the minus sign." This is important: it insures that obtaining the square root is always possible--insures that the expression is applicable to place-rich event-pairs (where Δx exceeds Δt) as well as to time-rich pairs. Note that L-interval is always positive.)

Example: If Able finds $\Delta t = 5$ ns and $\Delta x = 4$ ft, then he finds the L-interval to be:

$$\sqrt{\left|5^2 - 4^2\right|} = \sqrt{9} = 3.$$

Another example: If Able finds $\Delta t = 4$ ns and $\Delta x = 5$ ft, he finds the L-interval to be:

$$\sqrt{\left|4^2 - 5^2\right|} = \sqrt{9} = 3.$$

(Same result as above.)

In a general case, the events may be separated in all three space dimensions. Three space intervals are involved: Δx, Δy, and Δz. The full L-interval applies:

$$\text{L-interval} = \sqrt{\left|(\Delta t)^2 - \left((\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2\right)\right|} .$$

Frame-Invariance of L-Interval

The exciting fact is that, with respect to a given pair of events, the L-interval has the same value no matter what the frame of reference. Whether the measurements are made from the frame of the earth, or frame of a spaceship, or frame of an asteroid, the value of L-interval remains unchanged. Each frame has a different Δt and has a different Δx -- but all frames arrive at the same computed value of L-interval. L-interval is frame-invariant!

This is enormously helpful, as illustrated by the following example involving two lightning strikes. Judged from the Ableship frame, the strikes are separated by 135 ns and 100 ft. Judged from the Bakership frame, the time separation is 100 ns, and the space separation Δx is not known -- must be computed. The computation is simple. Because the L-interval must be the same for the two frames, the following must hold true:

$$\sqrt{(135)^2 - (100)^2} = \sqrt{(100)^2 - (\Delta x)^2}$$

Solving, we find: Δx = about 42.

Note: The whole subject of L-interval and its frame-invarance presupposes use of matched units, such as nanosecond and foot.

The Famous Surveyor Analogy

Consider a surveyor who is trying to find the distance between two igloos A and B on a large freely floating ice-cake that is being slowly rotated by wind and tide. On Monday he finds igloo A to be 5 miles east of B. On Tuesday it is 5 miles north of B. On Wednesday it is 3 miles east of B and 4 miles north of B.

Monday   Tueday   Wednesday
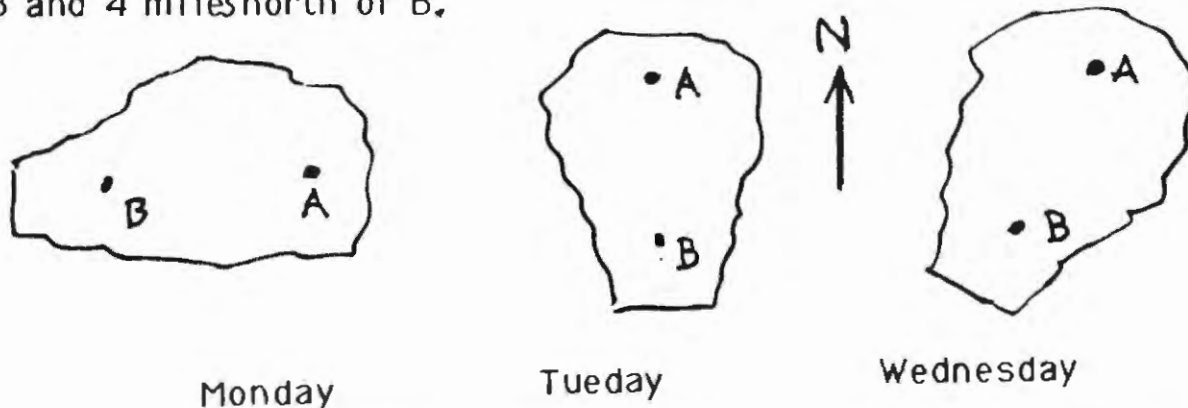
Does the surveyor find these results contradictory? Irreconcilable? Not at all. Each day, he computes the square root of the sum of the squares and ends up with the same number. For example, $\sqrt{5^2 + 0^2} = 5$, $\sqrt{3^2 + 4^2} = 5$. He says: "A is 5 miles from B." And he adds: "This is true no matter how the ice-cake has rotated."

Let us slightly revise the scenario. Assume that, at a given hour on a given day, several different surveyors make measurements -- each surveyor having a different idea as to which way is north--a different maladjusted compass. For the two igloos, one surveyor finds the north and east components of separation to be 5 miles and 0 miles, another surveyor finds 3 and 4, another finds 0 and 5.

Are the surveyors' computed distance values in disagreement? Not at all. Each arrives at the value 5 miles. Note that, because different reference grids are used, the "northness" values differ from one surveyor to another, and the "eastness" values likewise. But the resulting <u>distance</u> values are the same. They are "reference-grid" invariant. What is important is the distance, and this does not depend on the reference-grids' orientations.

<u>Crucial importance of matched units</u> If the surveyors always expressed northness in feet and eastness in yards, attempts to compute an invariant distance would fail. The data for sister dimensions can be combined effectively only if matched units are used.

The same applies to efforts to combine Δt and Δx in the L-interval. Unless matched units are used, the effort will fail.

<u>The analogy:</u>

What is important to the surveyors is distance, computed as the square root of the sum of squares. This distance is grid-invariant,

What is important to a person dealing with two events in spacetime is the L-interval (4-D distance), computed as the square root of a combination of squares. The L-interval is frame-invariant.

The analogy is sufficiently close that physicists regard all four quantities Δt, Δx, Δy, Δz as dimensions and as <u>belonging to one family.   -- One family comprising a four-dimensional spacetime.</u>

In particular, time is a <u>sister dimension to the familiar x,y,z dimensions.</u> It does not merely "exist independently along side of them" (as might be the case with, say, temperature or pressure) but is <u>integral with them</u>--coupled to them in a clear and exactly specified way.

The implications of this are enormous. No longer can we regard time and space as two very different and independent quantities. They are linked. They depend one on the other.

<u>Acknowledgment:</u> The surveyor analogy is featured with enormous success in the book "Spacetime Physics" by E. F. Taylor and J. T. Wheeler. The present writer is greatly indebted to those authors.

Limitation of the Analogy

As is already apparent to the reader, the analogy has a big flaw. The surveyor's formula contains a plus sign -- no minus sign. The northness term and eastness term. are squared and added. They work together. If either one is increased, the distance is increased.

But in the formula for L-interval there is a minus sign. The time-squared term and space-squared term are in opposition to one another. Thus for any given (Δt, Δx), the L-interval is always <u>less than</u> the larger of the quantities Δt, Δx. If, for a given event-pair, Δt and Δx are equal, the L-interval is zero.

For these reasons, the time dimension and space dimension might be called   "antagonistic sister" dimensions, Or each might be called an "anti-dimension" with respect to the other.

For sure, the minus sign has enormous philosophical implications (including a "hyperbolic space" implication!)

Range of Values of L-Interval

Obviously, each quantity Δt and Δx can range from 0 to infinity--but can never be negative.

L-interval likewise can range from 0 to infinity. It can never be negative--because it is defined as the square root of the <u>absolute value</u> of a difference.

Co-Change Rule

Consider a given pair of events and consider the Δt and Δx values found from a certain Frame A. If, from some Frame B, Δt is larger, then Δx also is larger. In other words a change of frame causes both quantities to wax or both to wane. Hence this rule:

<u>Co-Wax Co-Wane rule</u>: Changing frames causes Δt and Δx to co-wax or co-wane.

Relative Magnitudes of Change

It is easily shown that the following rules are valid. (They are corollaries of the rule that L-interval is frame-invariant.)

If a certain Frame A's values of Δt and Δx are almost equal, then the values found from a Frame B (a frame with respect to which Δt and Δx are larger) will be even closer to equality.

If a Frame A's value of Δt is very small and its value of Δx is large, then a frame change that leads to a greatly increased Δt will lead to only a slightly increased Δx. And vice versa. In other words, a big change in a small delta will be accompanied by a small change in the larger delta.

Bigger-and-Closer Rule: When a frame-change leads to larger Δt and Δx, these quantities become closer together.

Example: If Ablecrew finds Δt and Δx to be 115 and 57, some other crew might find: 709 and 703--close together!

These rules are illustrated in the following graph, applicable to one particular event-pair lying on the x-axis and evaluated from any and all frames traveling parallel to this axis.
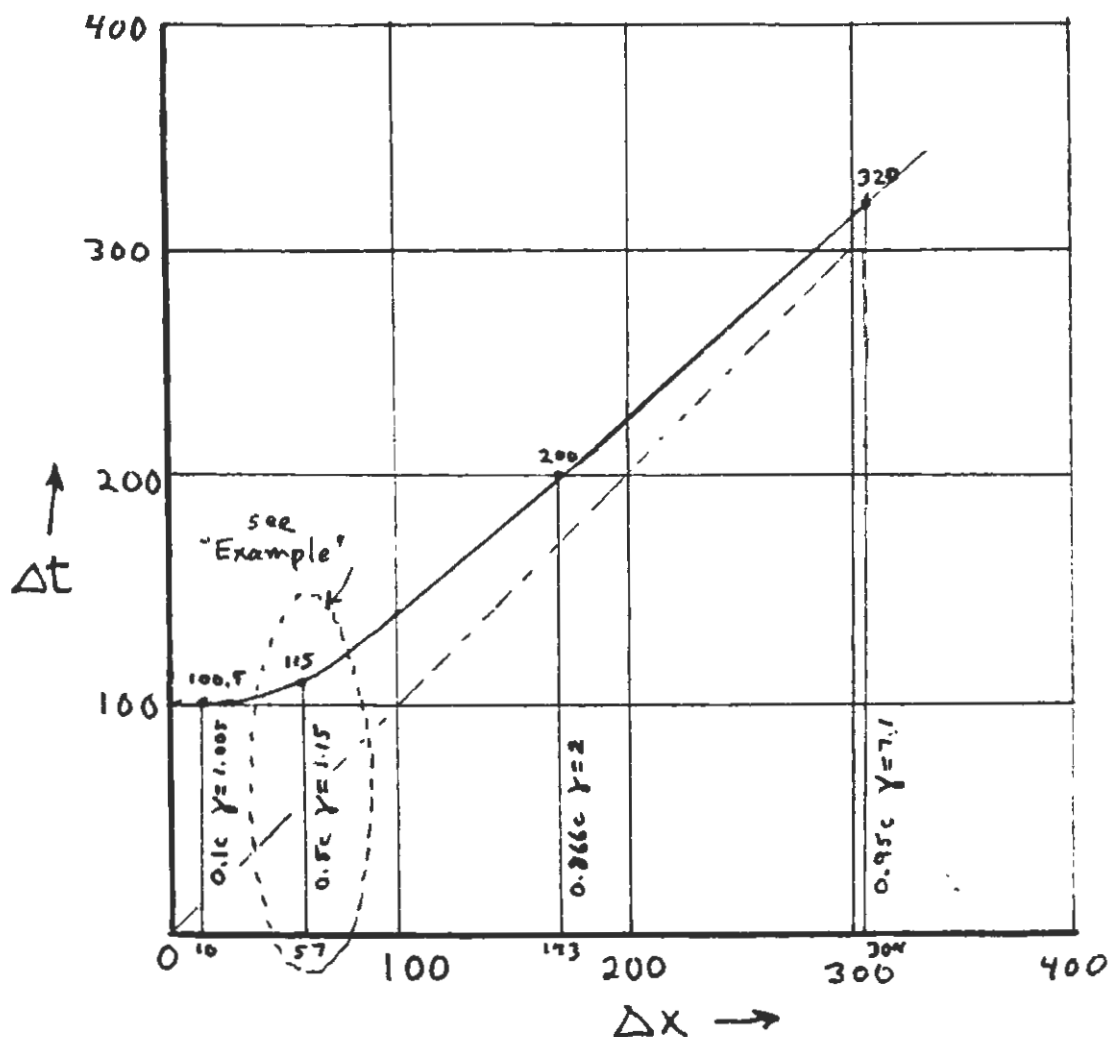
Graph of Δt vs. Δx for an event-pair for which, judged from a certain frame (nulling frame), Δt = 100 ns and Δx = 0 ft.

Example: Suppose that, with respect to this event-pair, Smith in his frame finds Δt to be 115 ns. What value of Δx will he find?   57 ft., indicated in left portion of the graph.

A similar graph applies to a place-rich event-pair. Regard the vertical axis as the Δx axis and the horizontal axis as the Δt axis.
Note: The curved line shown is part of a hyperbola and is asymptotic to a 45-deg. line.

Table of values: Consider Frames A, B, C, D, E that, relative to your frame, are at speeds 0.10 c, 0.50 c, 0.87 c, 0.95 c, 0.99 c. Suppose that, judged from your frame, $\Delta t = 100$, $\Delta x = 0$. Then with respect to the other frames, the intervals (deltas) are as follows:

Frame	Relative speed	Δt interval	Δx interval
yours	0	100	0
A	0.10 c	100.5	10
B	0.50 c	115	57
C	0.866 c	200	173
D	0.95 c	320	304
E	0.99 c	709	703

Convergence Rule

Inspection of the table and graph presented above shows the following rules to be valid:

- For the given event-pair, your frame is the only one with respect to which one of the deltas is zero. Also, this frame provides the greatest difference between the two deltas.
- The Δt and Δx values found from any other frame will be larger and closer together.
- The higher the speed of the other frame (relative to you), the smaller the difference between the deltas. That is, higher speed leads toward convergence.

Shorter Names: "Sumtor" and "Diftor"

Definition of the term "Sumtor"   A surveyor's verbal statement of the process of computing distance from the northness and eastness components Δy and Δx is long and tedious. He says the distance is "the square root of the sum of the square of Δy and the square of Δx." Complicated!

In this book I use the newly coined term "sumtor", thus: I say distance is "the sumtor of Δx and Δy."

Sumtor of Δx and Δy means: $\sqrt{(\Delta x)^2 + (\Delta y)^2}$.

Definition of the Term "Diftor"   Most textbook writers' verbal statements of the process of finding L-interval from the Δt and Δx components are long and tedious. They use the expression "square root of the magnitude of the difference of the square of Δt and the square of Δx." Complicated !

In this book I use the newly coined term "diftor", thus: I say L-interval is "the diftor of Δt and Δx."

Diftor of Δt and Δx means: $\sqrt{|(\Delta t)^2 - (\Delta x)^2|}$.

Example of use: Mr. Able finds, for two given events, time and place intervals of 4 ns and 3 ft. The L-interval is the diftor of these, or about 2.6. (Note: $\sqrt{4^2 - 3^2} = \sqrt{16 - 9} = \sqrt{7}$ = about 2.6.)

Another example: Given $\Delta t = 10.5$ and $\Delta x = 10$, the diftor is about 1.
(Note: $\sqrt{(10.5)^2 - (10)^2} =$ about $\sqrt{101 - 100} = 1$.

The term "sumtor" is easy to remember, since the first syllable, "sum", is a reminder that pertinent terms are to be summed. Likewise "diftor" is easily remembered; "dif" is a reminder that we deal with the difference of terms.   The syllable "tor" is familiar, as in factor, vector, operator.

The Unit of L-interval

If time and place are sister dimensions and are to be linked together by diftor process, their units must be the same. Does this mean that 1 ns and 1 ft are identical concepts? A nanosecond is a foot? Is a meter of place identical to a meter of time?

In a sense, Yes.

It would be convenient to have a unit that is free of bias --is historically tied neither to time nor to place.

In my writings I often use a specially coined term "spud". I use it as the unit of spacetime, representing 1 ns and/or 1 ft. I may say "the diftor of 4 ns and 3 ft. is about 2.6 spud."

"Spud" is an acronym for "Spacetime unit dimension",

It is amazing that no textbook authors use a neutral name for the unit of L-interval. They rely on avoidance tactics and circumlocutions.

Interpretation of L-Interval

Interpreting L-interval is a great challenge. Perhaps no fully adequate interpretation or understanding exists (just as we have no real explanation of the fact that the speed of light has the same value judged from all frames)

Here are three interpretations:

*1. L-interval is a kind of Rosetta stone. It helps us in trying to relate time and place intervals found from one frame to the time and place intervals found from another frame. For example, if we know the Ableframe Δt and Δx, and know the Bakerframe Δt only, we can at once calculate the Bakerframe Δx. If Able's Δt and Δx are 50 and 10, and Baker's Δt is 30, then we can find Baker's Δx merely by solving:

$$\text{Difftor } (\Delta t, \Delta x)_{Able} = \text{Difftor } (\Delta t, \Delta x)_{Baker}$$

$$\text{or } \sqrt{(50)^2 - (10)^2} = \sqrt{(30)^2 - (\Delta x)^2}.$$

*2. L-interval is the true 4-D metric, true "4-D distance" between events. 3-D distances found from a dozen different frames will differ--there will be a dozen different values. But the L-interval is frame-invariant. Each frame's data leads to the identical value. L-interval is an "all-frame" value --agreed on by observers in all frames. Thus it is a true distance, the only true distance, a true 4-D distance. Something unique, almost "sacred".

*3. L-interval is simply the minimized non-zero member of the $(\Delta t, \Delta x)$ set. Consider two events of a time-rich pair, two events that, judged from Ableframe, are found to be separated by $\Delta t = 5$ ns and $\Delta x = 3$ ft. Then there exists a frame (which I call the "nulling frame")*with respect to which $\Delta t = 4$ ns, $\Delta x = 0$ ft. (Proof: $\sqrt{5^2 - 3^2} = \sqrt{4^2 - 0^2}$.) Thus 4 ns is the minimized time interval between the two given events. <u>The L-interval of a time-rich event-pair is merely the minimized time interval.</u> Correspondingly, <u>the L-interval of a place-rich event-pair is merely the minimized place interval.</u> Thus L-interval is same as richness.
    Thus anyone who is computing the L-interval is merely computing the non-zero interval pertinent to the nulling frame. He is not computing anything new or exotic --but

* "Nulling frame" is discussed in detail in Chap. 6.

merely the simplest and most pedestrian non-zero member of the pair of deltas pertinent to the nulling frame.

As a corollary, he is merely <u>finding --with respect to all possible frames--the smallest value of the larger of Δt and Δx.</u> In the example given above, the 4 ns Δt-value is smaller than the Δt values found from all other frames.

<u>Comment re default by most textbooks:</u> I have been unable to find, in any textbook, a statement that L-interval is merely the non-zero member of the nulling-frame's (Δt, Δx) set-- is merely the smallest value of the larger of Δt and Δx.

Textbooks stress the above-stated interpretation #2 and fail to mention interpretation #3. Thus they leave the reader believing that L-interval is a mystic 4-D distance. They fail to say that it is an old friend, calculable (by diftor process) from any frame.

Of course, all three interpretations have much in common. For sure, it remains exciting and amazing that, knowing any frame's Δt and Δx, we can calculate the nulling-frame data.

Application of Reversed Form of L-Interval

A remarkable relationship arises if, for a given time-rich event-pair, one takes advantage of the equality of (a) the diftor of deltas found from some general frame (call it "Ableframe") and (b) the diftor of deltas found from the nulling frame (call if "nf"). Start with this equality

$$\sqrt{(\Delta t)^2_{Able} - (\Delta x)^2_{Able}} = \sqrt{(\Delta t)^2_{nf} - 0}$$

We can transpose it to this form:

$$(\Delta x)_{Able} = \sqrt{(\Delta t)^2_{Able} - (\Delta t)^2_{nf}}$$

This is a remarkable formula. It shows that Able's Δx can be regarded as the "residuum" between two time intervals-- can be regarded as a "time-interval excess": the square root of <u>one time interval squared minus another time interval squared.</u>

Amazing! A place interval is found by comparing time intervals!

What a dramatic demonstration of the linkage of time and place!

I have not seen this cleanly stated in any textbook.

Of course, a corresponding situation applies to an event-pair that is place-rich. If Able finds $(\Delta x)_{Able}$ to be 230 ft. and learns that the nulling frame's Δx is 200 ft., he can find $(\Delta t)_{Able}$ thus:

$$\Delta t_{Able} = \sqrt{230^2 - 200^2} = 114\,ns.$$

Amazing: A time interval can be found by comparing two place intervals! I don't recall seeing this fact stated in any textbook.

L-Interval Is an Old Friend: Richness

The definition of L-interval is the same as that of richness. If an observer finds, for a given pair or events, Δt and Δx of 115 ns and 100 ft., the L-interval is the diftor of these values --and the same applies to the richness, defined in Chapter 4.

Chapter 6
Special Frames and Special L-Intervals

Introduction

All of the exciting phenomena of special relativity occur with respect to frames. They are the stages on which the dramas unfold. This chapter explains some special names given to frames --names indicative of application.

As explained in Chap. 1, all inertial frames are on a par. There is no unique frame, no basic reference frame.

But with respect to a given physical object (spaceship, clock, etc.) or a given pair of events, a frame can be special.

There are two main classes of special frame: proper frame and nulling frame.

Proper Frame (Rest Frame)

A proper frame, or rest frame, is the frame in which an object under discussion is at rest. New York City's Empire State Bldg. is at rest in the frame defined by the grid of city streets.

Application to Length of an Object

Warning: In the following discussions I often enclose physical-property words such as "length", "rate", etc. in quotation remarks as a reminder that we are not dealing with true quantities but with quantities that are "eifo"; "eifo" is the acronym for "effective with respect to the indicated frame only." This subject is discussed in detail in a later chapter.

Consider Ableship and its proper frame. Measured by tools in this frame, the ship's length is, say, 100 ft. The fact is, measurements made by tools in any other frame B will yield a smaller number, smaller "length". (We assume the ship is oriented parallel to B's line of relative motion.) The greater B's relative speed, the smaller the resulting "length" figure.

Thus "length of an object" measured from the proper frame is an extreme with respect to all frames. It is

unique. It is the greatest of the "length" values. In other
words, the proper frame is the "length maximizing frame."
This applies to lengths of spaceships, tennis courts,
metersticks, meteorites, mice, etc.

Application to Length of Period of a Cycle

Consider a child's swing that executes one complete
to-and-fro motion every two seconds--a swing whose
period is 2 seconds, measured by tools in the proper frame
(the playground, say). The fact is, measurements made by
tools in any other frame B will yield a larger number --
longer "period".

Thus "period of a cycle" measured from the proper
frame is extreme with respect to all frames. It is unique.
It is the smallest of the "period" values. In other words,
the proper frame of a cycling object is the "period
minimizing frame." This applies to periods of swings,
violin strings, clock pendulums, and the monthly orbiting
of the moon.

(The fact that results found in proper frames show
maximizing for length (of tennis court for example) but
minimizing for period (of a swing for example) may seem
surprising. It is discussed in a later section.)

Note concerning inverse of length

Instead of focusing on an object's length, we could
focus on the reciprocal, or inverse, quantity: number of
such objects per unit length. Consider, for example a
tightly-packed row of 3-in.-diameter oranges, with four
oranges per foot--all as measured by tools fixed in the
same frame with the oranges.

Fact is, measured from any other frame, the "length"
of an orange is smaller but the "number of oranges per
inch" is larger.

> Analogy involving light: one speaks of wavelength (length of one
> wave) and frequency (number of waves per sec.). Both
> concepts are useful, but one must keep clear which is the one
> under discussion.

Note concerning inverse of period

Consider the above-mentioned child's swing. Instead
of dealing with the swing's period, we could deal with the
number of cycles per second, that is, the frequency, or

rate. If the period is two seconds, the frequency, or rate, is 1/2 cycle per second (1/2 cps).

Fact is, measured from tools fixed in any other frame, the "frequency" ("rate") of the child's swing is less. The proper frame is the one whose tools maximize the "frequency" ("rate").

Graph concerning "length": The following graph suggests how the number (pertinent to the above-discussed "length") varies with the speed of the improper frame relative to the proper frame. As one considers successively higher speed (speed in either direction) one finds the "length" to decrease toward zero.
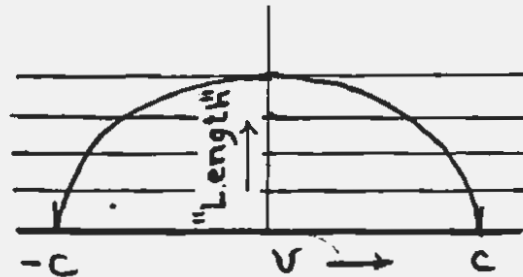
A similar graph applies to frequency (rate) of a child's swing or other cycling object. The higher the relative speed of the improper frame, the lower the frequency (rate).

Consider now the inverse of length. The following graph shows how the "number of objects per foot" changes as measurements are made from improper frames at successively higher speed relative to the proper frame. The values increase without limit.
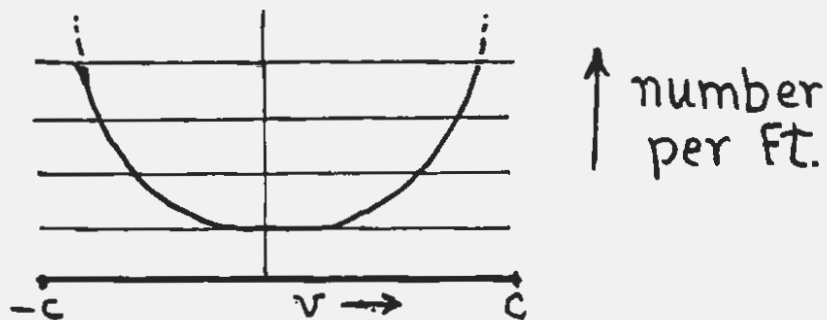
A similar graph applies to the period of a cycling object.

Why Consider Inverse Quantities?

For two reasons:

(1) To avoid confusion. To recognize that quantities have inverses--and that one must keep clear as to whether one is dealing with the direct quantity of the inverse quantity.

(2) To keep alert to the major mismatch between the commonly used space term and the commonly used time term:

Dealing with space, we use "length", a <u>direct</u> term,

Dealing with time, we usually use frequency ("rate"), an <u>inverse</u> term.

The effect of this mismatch is discussed in a later section.

There is the further complication that many writers use the term "time dilation". What is dilated (increased) by the relative speed of the improper frame is the period, not the rate. What is dilated is the direct quantity, not its inverse.

What is said above concerning length of a ship or length of an orange applies equally to the length of a foot ruler--and to "length of a foot". Ablecrew finds that Bakership's foot ruler is "short", and finds that more than three Bakership foot-rulers will fit within an Ableship yard.

Likewise what is said above concerning period of a (single) swing applies equally to the period of a (single) clock. Ablecrew finds that one second indicated by Bakership's clock is "long" and fewer Bakership seconds fit within an Ableship hour.

"Proper" Not Applicable to Pair of Events

The term "proper frame" does not apply to a pair of events. An event resides in all frames--has no allegiance to any one frame or any one object. It has--ideally--no length, no duration. Also, the two events of a given pair may be entirely unrelated.

For these reasons, it is unwise to impute to an event-pair a proper frame.

Nevertheless, special purpose frames can be identified, as explained below.

Nulling Frame

With respect to a given pair of events, the nulling frame is the frame whose tools show the <u>time interval</u> or <u>place interval</u> to be null (zero).

If the event pair is time-rich, the nulling frame's tools show $\Delta x = 0$. If the event pair is place-rich, the nulling frame's tools show $\Delta t = 0$.

Also, if the event pair is time-rich, the nulling frame's tools minimize the Δt value; all other frames' tools yield larger values. If the event pair is place-rich, the nulling frame's tools minimize the Δx value; all other frames' tools yield larger values.

Thus with respect to a given time-rich pair, a certain frame may be called "nulling frame" or "Δx-nulling frame" - -or may be called "Δt-minimizing frame". With respect to a given place-rich pair, a certain frame may be called "nulling frame" or "Δt-nulling frame" -- or may be called "Δx-minimizing frame".

<u>Coplacey Frame (or Coplacing Frame), Cotimey Frame (or Cotiming Frame).</u> These terms are sometimes used to mean the Δx-nulling frame and the Δt-nulling frame.

<u>Example involving a time-rich pair</u> In my laboratory a short-circuit occurs in my desk-lamp and a fuse blows out 115 ns later in a utility room 57 ft. to the east. Thus with respect to the laboratory frame, $\Delta t = 115$ ns and $\Delta x = 57$ ft. But with respect to a spaceship passing by in eastward direction at relative speed 0.87 c, the time interval and place interval are smaller: they are $\Delta t = 100$ ns and $\Delta x = 0$ ft. Thus the spaceship defines the frame with respect to which the two events are coplacey--the nulling frame of the event-pair, (Note: the diftor of 115 and 57 is 100 and the diftor of 100 and 0 likewise is 100.)

<u>Example involving a place-rich pair</u> Lightning strikes the south end of a 300-ft.-long football field at noon, and another strike hits the north end at 260 ns past noon. With respect to observers on the field, $\Delta x = 300$ ft. and $\Delta t = 260$ ns. With respect to a spaceship speeding north at 0.87c, the intervals are $\Delta x = 150$ ft., $\Delta t = 0$ ns. (The diftor of 300 and 260 equals the diftor of 150 and 0.) Thus the spaceship defines the cotimey frame: the event-pair's nulling frame.

Relationship to L-Interval

For a given time-rich event-pair, the minimized Δx is simply the pair's L-interval.

Likewise for a given place-rich event-pair, the minimized Δt is simply the L-interval.

Conclusion

When you have identified the nulling frame and found the minimized delta (the non-zero delta), you have found the L-interval.

Finding the Nulling Frame

For a time-rich event-pair and for a place-rich event-pair the procedures for finding the nulling frame are nearly alike.

Time-Rich Pair

Suppose Able finds, for a certain time-rich event-pair, Δt = 115 ns and Δx = 100 ft. Assume the more easterly event occurred last. How, exactly, is the nulling frame to be specified? Answer: by specifying its travel direction relative to Able and its speed relative to Able.

In this particular case, the nulling frame's travel direction relative to Able is east. Obviously, only an eastward direction can lead to a diminished Δx.

The speed relative to Able is easily found--in three steps.

Step 1: Find the proper Δt. It is the diftor of 115 and 100, namely $\sqrt{[(115)^2 - (100)^2]}$ = 57 ns.

Step 2: Find the ratio of Δt values: ratio of Able's Δt to the   proper Δt. In the present case the ratio is 115/57 or about 2. That is, gamma = about 2. (Gamma is discussed in Chap. 9)

Step 3: Find what relative speed implies this ratio. In the following equation, solve for v/c.

$$2 = \frac{1}{\sqrt{1 - (v/c)^2}}$$ Answer: v/c = 0.87.

--or simply refer to the graph or table shown in Chapter 5.
. One finds: relative speed = 0.87c.

Place-rich pair

For a place-rich event-pair the procedure is nearly the same. If Able finds the more easterly event to have occurred last, the proper frame's relative direction of travel must be east, as required by the "FF rule" of Chap. 3. To find the nulling frame's relative speed, find the minimized place interval, then find gamma (the ratio of Able's place interval to minimized place interval), and then find what speed gives this gamma.

Two Options for Transframe Comparisons of Clocks

Option 1: The option usually assumed (when Ablecrew wishes to measure the performance of Bakership's clocks) is for Ablecrew to employ two synced Ableship clocks and record their readings as a single Bakership clock passes by--and record the Bakership clock's readings. Notice the asymmetry of clock use: two clocks in Ableship, one in Bakership. (See diagram.) As explained elsewhere (see "Co=small rule", Chap. 7) the outcome is that the difference in readings of the single Bakership clock is less than the difference in readings of the two Ableship clocks. That is, the Bakership clock is found to be "slow" (more exactly, eifo-slow) compared to the Ableship clocks.
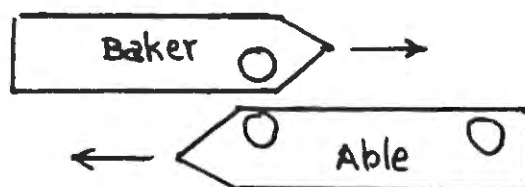
Option 2: Option 2 is for the Ablecrew to use a single Able clock and record its readings as two spaced clocks in Bakership pass by--and record those clocks' readings also. (See diagram.) The result is that the Bakership clocks are found to be "fast" (more exactly, eifso-fast) relative to the Ableship clocks.
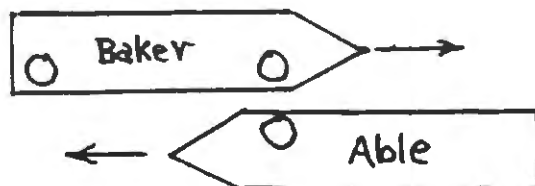
Which option is correct?

Option 1 has the merit that the Able crew does not have to trust another frame's sync. The synced pair of clocks is in Able ship itself. This is the option most testbook-writers assume.

Option 2 has a possible "intellectual merit" -- the merit that the scenario parallels the scenario used in measuring length. Here is the parallelism: In Ablecrew's length measurement of Bakership, Ablecrew makes two position determinations (impinging,say, on the two ends of Bakership) at the same per-Ableframe instant. That is, the measurements are Ableframe-cotimey. Therefore the parallel scenario regarding clocks is for Ablecrew to make the measurements at the same per-Ableframe place. That is, make measurements that are Ableframe-coplacey.

The two options give opposite results. Option 1 implies Bakership clocks run eifo-slow, and Option 2 implies they run eifo-fast.

If this latter option is adopted, the implication is that the Bakership clock period is eifo-small --paralleling the conclusion found for the Bakership length, also eifo-small. "Small" applies to both quantities--the sister quantities length and period. It is intellectually satisfying that the outcomes for the two quantities (for the two dimensions, space and time) are parallel.

Parallel scenarios, parallel outcomes! Great!

(If we adopt Option 1, we find opposite, i.e., antiparallel, outcomes. Disturbing!)

It is surprising that no textbooks mention the two options. They use Option 1 with no explanation -- as if no other option were available. Also, they seem not to notice the anti-parallelism of the outcomes.

Chapter 7: Interframe Comparisons of Clock Readings, Basic Facts

Introduction

As indicated in earlier chapters, the heart of special relativity is interframe comparisons, such as the comparison of Ableship data and Bakership data concerning, say, lightning strikes in two suburbs of Boston, or the length of a certain spaceship, or length of time between two successive ticks of a clock, or the speed of a meteorite, or the momentum of a bullet, or the lifetime of a muon. Usually we deal with comparisons of intervals between events: time intervals and place intervals.

Chapter 5 dealt with time intervals between events. It dealt with events such as lightning strikes, collisions, pistol shots, laser light pulses, etc. --short-time small-area happenings.

The present chapter also deals with time, but with a special focus: comparisons of readings of clocks in different frames or, equivalently, comparisons of the durations of seconds in different frames. Our focus is on the simplest and most interesting case: one frame contains two synced clocks and the other frame contains a single clock. The frames pass by one another at high relative speed, and the single clock passes by the two clocks. Whenever a "clock passing by" occurs, the clock readings are noted and the disparities of readings are examined.

Topics Postponed to a Later Chapter We postpone answering the question: "What, ultimately, causes the disparities of readings? The clocks themselves? The relative speed? The observers? Spacetime itself?" Or is there, indeed, any cause?

This question is examined in detail in Chapter 15.

Case of Principal Interest

Suppose the spaceship Ableship contains two clocks, A_1 and A_2, which are spaced some distance apart along Ableship and have been synced in the Ableship frame. Suppose Bakership contains a single clock, B_1. Suppose, also, the ships pass close by one another, so that B_1 passes successively past A_1 and A_2. At each of these passings-by, the clock readings are recorded.
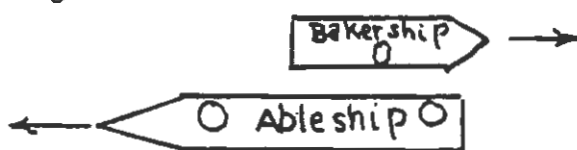
Drawing made by Able

The main fact: The difference between the two readings of the Able clocks exceeds the difference between the two readings of the single Baker clock.

In other words, $(\Delta t)_{Able}$ exceeds $(\Delta t)_{Baker}$. briefly: $(\Delta t)_{Able} > (\Delta t)_{Baker}$. Equally well we can say: the one-clock Δt is less than the two-clock Δt. The alternative statements employing "exceeds" or "less than" are, of course, equivalent.

This above-stated main fact can be proved by pure reasoning (see next page) or proved by actual measurement. But first, an important assumption concerning the ruggedness of clocks must be presented.

Clocks Themselves Not at Issue

We assume, of course, that all clocks are of high quality and are properly operated. They are so rugged that even if, in the past, they have been accelerated fairly vigorously, they continue to function normally. Even if they were accelerated and traveled at high speed to the moon and back, when back on earth they would keep time just as they did initially.

The effects we are considering--effects considered in

all textbooks on special relativity--have nothing to do with the physical conditions of the clocks but depend solely on the method, or basis, of comparison (discussed in a later section). Indeed, the effects in question are <u>contingent on the clocks' performing entirely normally.</u>

Prelude to Proof

Suppose a big 10-ft-high truck is traveling along close beside a high brick wall at a speed of 40 ft/sec, and suppose a man on the truck throws an ideally elastic rubber ball straight downward (relative to the truck) at about 40 ft/sec. (relative to the truck). Then it will bounce upward back to his hand in 1/4 + 1/4 = 1/2 sec. This is the round-trip time. (I neglect slight changes in speed due to gravity.)
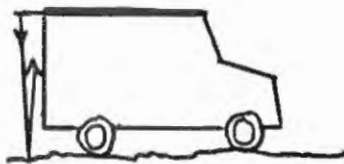
As judged by man on truck   As judged by children
                                              on wall
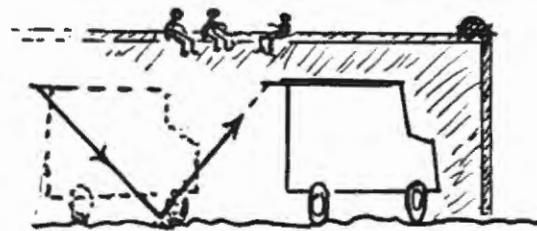
Now consider a long row of children seated on the wall. Judged by them the ball travels at an angle of 45 degrees and has speed much greater than 40 ft/sec. The ball has a downward component of speed (40 ft/sec) and a horizontal component (40 ft./sec), giving a total speed of $\sqrt{40^2 + 40^2}$ or about 56 ft/sec. The distance traveled (at the 45 degree angle) is $2(\sqrt{10^2 + 10^2})$ or about 2 x 14 ft., or 28 ft. So what is the duration of the ball's complete trip?   Distance divided by speed, or 28 ft./(56 ft./sec.), or 1/2 sec.

<u>Summary:</u> In this scenario, where only very low speeds are involved, persons in different frames find different ball speeds and different travel distances, but the <u>same travel time</u>.

All this is straightforward -- and not interesting. But it sets the stage for the crucially different scenario presented below.

Proof Relying on the Basic Laws

We now arrive at one of the grandest, simplest, and most convincing proofs of special relativity--based on the laws presented in Chapter 2.

Suppose that the long slender spaceships Ableship and Bakership are passing close by one another at high relative speed, specifically, 87% of c (see following diagram). Suppose they have smooth shiny surfaces. Suppose they pass within 10 ft. of one another. Suppose, finally, that a light source on Bakership sends a short pulse of light aimed exactly transverse to Bakership and toward Ableship (all as judged by Bakercrew).

The light pulse is reflected from Ableship and returns to Bakership and is there detected.

According to Bakercrew, the round-trip distance is 20 ft. and necessarily the round-trip travel time is 20 ns (since light travels 1 ft/ns. judged from any frame--per Law #5).

According to Ablecrew, the light pulse follows a two-segment slanting path (see sketch) and advances along each segment at speed c (since light travels at speed c judged from any frame). The component of motion parallel to the line of relative motion has the speed 87% of c, according to our initial assumption. The distance between the two ships is 10 ft., also according to our initial assumption.
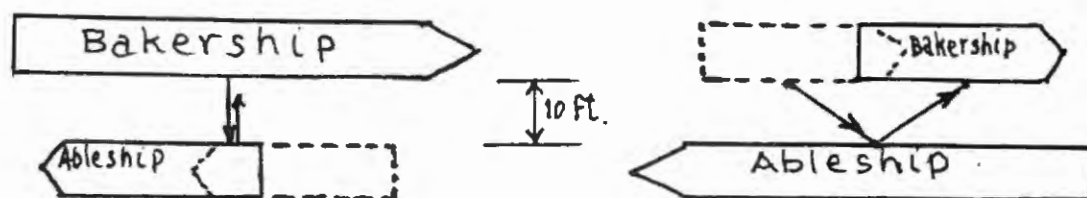
As judged by Bakership crew   As judged by Ableship crew

Let us find the length of one slant path of the light pulse-- one hypotenuse. Call its length x. Then we deal with a triangle whose hypotenuse is x and whose vertical leg is 10 ft. What is the length of the horizontal leg? 0.87x, because the relative speed of Bakership is 0.87 times the speed of light.

Using the Pythagorean law, we write

$$x^2 = (10)^2 + (0.87\ x)^2 = 100 + 0.75\ x^2$$

or $0.25\ x^2 = 100$

or $x^2 = 400$

or, finally, $x = 20$ ft.

One segment of light-pulse path -- x ≈ 20 ft.
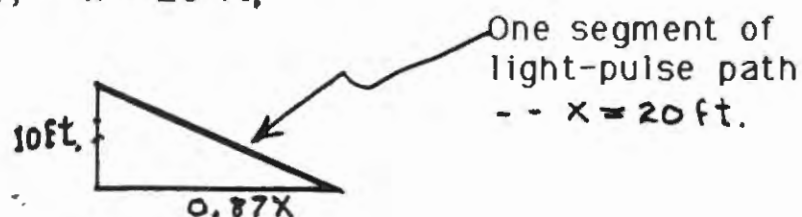
Thus the overall pathlength, according to Ablecrew, is 20 + 20 = 40 ft., and necessarily the travel time --the Δt -- is 40 ns (because the speed of light is 1 ft/ns judged from every frame.)
Thus our result: Ablecrew's Δt is twice Bakercrew's.

Summary: The time-interval Δt indicated by the Ableship synced clocks is twice the time-interval Δt indicated by the Bakership clock.
Incredible? No. Each step in the proof is simple and clean cut. See for yourself!

Proof Employing Experimental Confirmation

In the above scenario, Ablecrew computed the light-pulse pathlength (40 ft.), then <u>calculated</u> the travel time (40 ns) by employing Law #5 (light travels 1 ft. in 1 ns). Can this crew actually measure the travel time? Can it obtain the 40 ns result without having to rely on Law #5?

Yes -- if we make certain suppositions concerning clocks.

Assume that <u>Bakership</u> contains a single clock, B_1, that is immediately adjacent to the Bakership light source and detector --and records the time of emission of the pulse and the time of reception of the returning pulse. Assume that <u>Ableship</u> has two masts that project toward the Bakership line-of-motion and are almost 10 ft. long. At the top of each mast is a light-pulse detector and a recording clock. The clocks, called A_1 and A_2 (built by the same company that built Baker clock B_1) have been synced in the Ableship frame. The masts are situated just far enough apart so that one happens to be very close to Bakership's light source while this source is being pulsed and the other happens to be very close to Bakership's detector while it is detecting the returning pulse. See diagram,
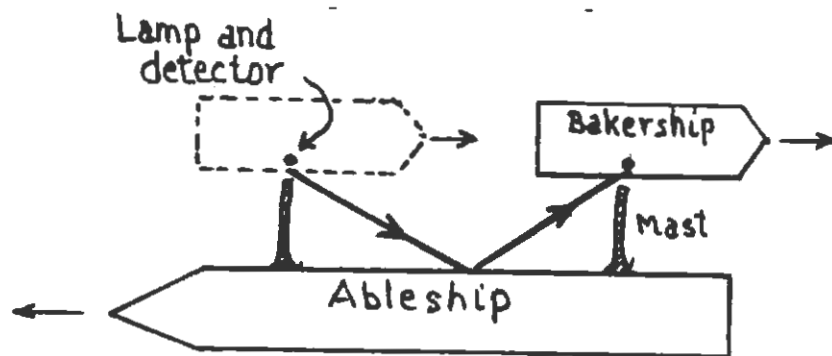
Diagram by Able
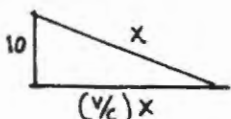
Using this pair of clocks, Ablecrew measures the times of start and finish of the pulse's travel. The difference in reading is the Ablecrew result for the travel time. Result: 40 ns.

General Case

Suppose, in the above scenario, the relative speed is called v.

Then, as before, the Bakership crew finds $\Delta t = 20$ ns.

What does Ablecrew find? It finds the light pulse's round-trip pathlength to be

$$10\left(\frac{1}{\sqrt{1-(v/c)^2}}\right).$$

Proof:

$$x^2 = 10^2 + (v/c)^2 x^2$$
$$x^2(1-(v/c)^2) = 10^2$$
$$X = \frac{10}{\sqrt{1-(v/c)^2}}$$

and accordingly it finds, for the round-trip travel time,

$$\Delta t = 20\left(\frac{1}{\sqrt{1-(v/c)^2}}\right).$$

What is the ratio of the two Δt's?

$$\frac{20\left(\frac{1}{\sqrt{1-(v/c)^2}}\right)}{20}.$$

Accordingly:

$$\Delta t_{able}/\Delta t_{baker} = \frac{1}{\sqrt{1-(v/c)^2}}.$$

The ratio is called "gamma", γ , a famous parameter of special relativity. It is discussed in detail in Chapter 9.

Crucial Asymmetry

In the above scenario, there is a crucial asymmetry: the asymmetry of the sets of clocks used.

<u>Bakercrew</u> measures the times of start and finish of the pulse's round-trip by means of a single clock. Judged by this crew, the single clock is present where the pulse starts and also where it finishes. Simple! --Especially as, there being but one clock, the subject of synchronization does not arise.

Ablecrew needs two clocks: one situated close to where the light pulse originates and the other close to where it terminates. These two clocks must, of course, have previously been synced.--synced with respect to the Ableship frame.

General conclusion: When a single clock passes by two spaced and synced clocks, the single clock's Δt is smaller than that of the pair of clocks.

What is the immediate cause of the difference between the two crews' Δt values? The asymmetry with respect to sync. The sync is provided by Able's clocks, not Baker's.(Causality is examined in detail in Chapter 15.)

Asymmetry of sync is the basis of the "Co=small rule" presented below.

Note: Obviously, if the transversely aimed pulse had been sent-and-received by Ablecrew (Ablecrew using one clock and Bakercrew using two), Ablecrew would have found the smaller Δt.

The "Co=Small Rule"

The rule is: "The time interval found in the frame in which the two measurements are coplacey provides the smaller time interval."
More briefly: "Coplacing-frame data provide smaller Δt."

Notice that the rule follows directly from Law #2: the speed of light is the same judged from all frames.

Simpler (Qualitative) Proof of the Front First (FF) Rule

A detailed quantitative proof of the FF (Front First) Rule has been presented in an earlier chapter. The proof is somewhat complicated and artificial, involving two spaceships equipped with special masts.

Here is an especially simple proof: a _qualitative_ proof involving nothing more complicated that a one-car train.

Suppose Able, standing near a railroad track, sees a one-car train passing by eastward at high speed. Suppose that two lightning strikes occur: the west strike hits the rear of the train and an adjacent tree, and the east strike hits the front of the train and an adjacent tree. Suppose Able is midway between the trees and receives the flashes from the two lightning strikes at exactly noon. He says: "I am midway between the trees, I have received the flashes at the same instant, and both flashes traveled at the same speed. I conclude that the flashes were cotimey."

Able
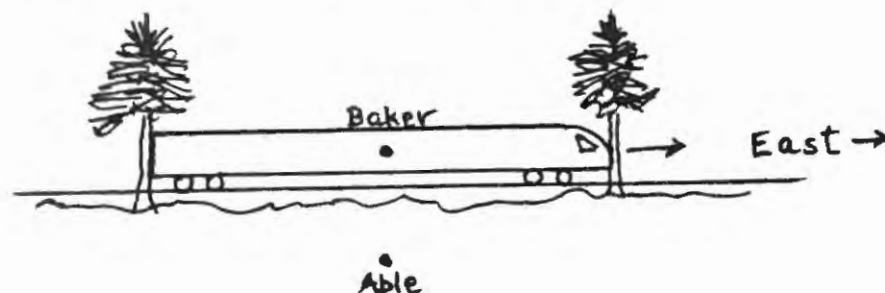
What is the finding by Baker, seated exactly in the middle of the train? While the two flashes are traveling toward him (from front of train and rear of train), he is moving eastward, toward the front strike. Therefore the flash from the front will reach him first. He says: "I am midway between the front and rear of the train, I received the front flash first, and both flashes traveled _at the same speed_, I conclude that the front flash occurred before the rear one---_front flash first_."

The scenario is especially reliable. Each lightning strike has produced a scar on the train and on the tree--permanent evidence that can be reviewed by both men.

Chapter 8: Interframe Clock Comparisons: Interpretations and Proposed New Term "Eifo"

Introduction

The previous chapter showed that when Able's two spaced and synced clocks pass successively by Baker's one clock, or (stated other way around) Baker's one clock passes successively by Able's two clocks, the Able-clocks' time interval between the two events (two passings-by) exceeds the Baker-clock time interval. That is,

$$(\Delta t)_{\text{Able two clocks}} \;>\; (\Delta t)_{\text{Baker one clock}}$$
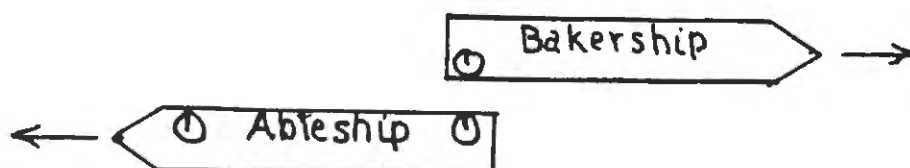
How is this result--this disparity -- to be interpreted?

Below, I discuss some bad interpretations, then some better ones. Then I show that, in a sense, what is needed is a new term--new prefix: "eifo".

In later sections I discuss the underlying cause of the disparity and the range of situations where it applies.

Bad Interpretation

A bad interpretation of Baker clock's smaller Δt is: "The Baker clock is at high speed relative to Able, and the high speed causes the clock to run slow. High speed slows a clock's rate."

Most textbooks endorse this interpretation, either directly, or by implication, or by default. It is a short explanation. Dramatic. Memorable. But wrong.

See Chapter 16 for a great many reasons why the interpretation is wrong.

The heart of special relativity is the equations. They do not contains words such as "slow" or "slowed". When a writer introduces these words, he is straying beyond the facts.

Attempts at Rebuttal

Someone may say: "There must be a slowing! Consider the short-lived (10^{-6} sec) <u>muon</u> particles produced in cyclotrons and synchrotrons. It is a well-proven fact that these particles, when at low speed, disintegrate very quickly, but when traveling at relativistic speed they endure longer--5 or 10 times as long, typically. Surely this implies 'clock slowing' since an unstable muon is a kind of clock."

Someone may say: "Consider a twin who whizzes to a star and back, and ends up younger than his stay-at-home twin brother. Consider the fact that the traveler's clock reads far behind the stay-at home twin's clock."

Overall contention: "There is a very real phenomenon here. It cannot be dismissed."

There is indeed a very real phenomenon.

What is the correct interpretation?

Correct interpretation

Suppose Ableship has ten clocks --uniformly spaced 8.7 ft. apart and synced--and Bakership's single clock whizzes very close past at 0.87c (for which gamma = 2).

Bakership O →

← ⟨ O O O O O O O O O O |
Ableship
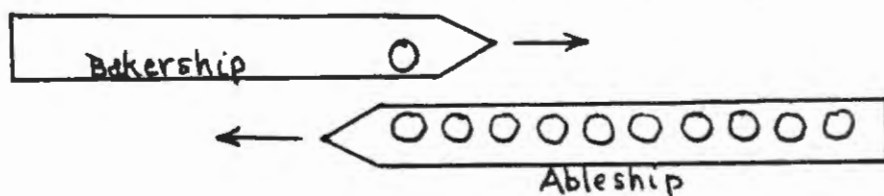
The single Baker clock travels successively past the ten Able clocks, and at each such passing-by, the readings of the two passing clocks are noted.

Suppose that, when the Baker clock passes by the first Able clock, both of these clocks read 0. Suppose the nine successive passings-by of clocks occur, according to the set of Able clocks, at 10, 20, 30, etc. ns, and they occur according to the <u>single</u> Baker clock at 5, 10, 15, etc, ns.

Thus it is strictly accurate to say:
> The readings of the Able clocks are successively
> more ahead of the readings of the (single) Baker
> clock. Stated the other way around: the readings of
> the (single) <u>Baker</u> clock are successively more
> <u>behind</u> the readings of the Able clocks.

<u>Summary:</u> It is clearly and absolutely true that, at the
successive passings-by, the readings of the Baker
clock exhibit behindness--successively increasing
behindness.
> Behindness of the Baker clock is a fact.

What about the converse? Can we say that the Able
clocks become successively ahead? Each Able clock exhibits
increasing aheadness?   No, because each such clock is read
only once. Accordingly, for the individual clock, no trend
is indicated. (The situation is different for Baker's clock: its
reading is noted again and again, and a trend --increasing
behindness ----is clearly apparent.)

True Cause of Behindness
In our daily lives, we encounter two causes of a given
clock's reading increasingly behind a standard clock:
> Cause 1: The given clock is defective--
physically impaired. It is truly running slow.
> Cause 2: The basis of comparison has changed.

This second cause applies to clock behindedness.

Think of this analogy: A Californian flies to Boston.
As he leaves California, his watch agrees with the airport
clock. But on arriving at Boston, he finds his watch is
three hours behind the airport clock. The behindness stems
from the change in basis of comparison-- change from
Pacific Standard Time to Eastern Standard Time.

It is this second type of cause that explains why
Baker's "one-clock" Δt values become progressively more
behind Able's "many-clock" Δt values.

When an observer team employs two clocks (situated
some distance apart) and has synchronized them,
 this sync is unique. It disagrees with syncs in all other
frames.

If Bakership passes by ten differently speeding spaceships each of which has a set of synced clocks, those sets have ten different syncs. Accordingly the time intervals shown by Baker's single clock will have ten different extents of behindness, relative to intervals shown by the ten sets.

Summary: The cause of the ten different extents of behindness is the ten different bases of comparison-- the ten different syncs.
The clocks themselves are in no way affected, not slowed, not speeded up. The cause resides in the comparison method, not any clock.
To use a crude analogy: clock behindness, like beauty, lies in the eyes of the beholder.

Exception: If the clock changes frame, there is a different cause of behindness. See a later section.

Both frames' Δt values are correct

Any notion that Ablecrew's Δt values are correct and the Bakercrew values are wrong --or vice versa--should be dismissed.

Each frame's values are correct. There is no contradiction, no conflict.

Each set of values is correct with respect to the pertinent sync.

The values are not correct in an absolute sense!   Just: correct with respect to the indicated sync. All observers in dozens of other frames (thus relying on dozens of different syncs) would obtain different values. All would disagree, for example, with the above-discussed 0, 10, 20 etc. values and 0, 5, 10, etc. values. These sets of values have no general standing. They do not command "all-frame respect". are valid for the indicated frame (indicated sync) only.

Key New Term: Eifo

To designate various special-relativity quantities that have no general standing, command no all-frame respect--yet are the result of accurate measurement from a given frame and are important to observers in this frame, I have introduced a new term,

The new term, or prefix, is _eifo_. It is the acronym for "_e_ffective for the _i_ndicated _f_rame _o_nly."

I have found the term to be enormously helpful: shortening and clarifying key statements.

It is applicable to a wide range of topics involving time, length, mass, speed, and other computed properties of physical objects. It does not apply to raw data such as Δt and Δx values.

Some examples of use:

If Ablecrew obtains results suggestive of "slowing" of Bakership's clocks, I may say that Ablecrew finds the Bakership clocks to be eifo-slow. I may say the high relative speed has an eifo-slowing effect.

If Bakership's rest length is 100 ft, and Ablecrew measurements yield the value 50 ft., I may say that, judged by Ablecrew, Bakership has the eifo-length 50 ft. The greater the relative speed, the greater the eifo-shortening.

Readers who become familiar with the new term will find it simplifies discussions, clarifies understanding.

Note: The term is dramatically useful in discussions of the pole-and-barn paradox, presented in Chapter 19.

Dereliction of Textbook Authors: No (?) textbook authors have introduced a term fulfilling the role of "eifo".

Some use this circumlocution: "Ablecrew _measures_ Bakership to be short". (What does this mean? Are the measurements right or wrong? Are we to understand that the ship is indeed short and measurements confirm this? Or are we being given a hint that the measurements may be wrong?)

Some experts use the words slow and "short" and place them between quotation marks--as some kind of a warning that something may be amiss.

Some experts say the Ablecrew findings "are _as if_ Bakership were short." Meaning, perhaps, that someone ignorant of special relativity would infer shortness but a more sophisticated person would not.

Worse yet: many authors insist that the slowness and shortness are straightforwardly real.

I believe "eifo" can plays a central role to improving discussions of special relativity.

Note: Many examples of textbooks writers' poor statements are presented in Chapter 16.

Note on Two Kinds of Grand and Valid Causes of Behindness

Both causes ultimately stem, of course, from the linkage of time and space. But this is vague. What is the more immediate and explicit cause?

There are two kinds of causes, applicable to two different kinds of scenarios.

Cause 1: Sync-Asymmetry

This kind of cause applies if each observer team always remains inertial -- never accelerates (never changes frame).

The cause: The heart of the cause is asymmetry with respect to clock use. One frame has spaced and synced clocks. In the other frame, there is only one clock and therefore this frame's sync is not defined, not used, not relevant.

Because of the asymmetry of sync, the two observer teams obtain, for time-interval between the two given events, different values. (See detailed proof in Chapter 7.)

Cause 2: "More Travel Means Less Time"

This kind of cause applies if, between the two given events, one observer team (Team B) undergoes a frame change and the other team (Team A) does not. (The cause applies, par excellence, to the twin paradox, presented in Chapter 20.)

Sync is not involved, and indeed there is no need for either frame to have more than one clock.

The cause: The heart of the cause is the relationship between travel and time. Here "travel" means distance Team B travels between two specified events--distance being defined with respect to Team A. The distance may include "back and forth" segments, or travel in a circle, or other travel involving change-of-frame.

"Time" means the between-events overall time interval indicated by B's clock--an interval that may be the sum of two or more intervals associated with the two or more frames occupied sequentially by B. The overall time interval is usually called proper time, or cumulative proper time t_{CP}.

The key fact is this: B's t_{CP} is less than A's.

This fact follows from the frame-to-frame invariance of L-interval; see Chapter 5. Also, it is a kind of rewording of the Co=small rule; see Chapter 7.

An ever better line-of-argument here may be to ignore rules and rely on direct experimental proof. Various high-energy physics laboratories have demonstrated again and again that a bunch of the short-lived muon particles has a longer half-life (implying a smaller t_{cp}) the greater its speed relative to the laboratory. Larger half-life correlates with smaller t_{cp}, and greater speed correlates with greater travel distance.

Such measurments on the effect of relative speed on a bunch of muons are in the same class with measurements on the speed of light. Both kinds of measurements demonstrate the linkage of time and space.

The demonstrations -- the facts--do not depend on any theory ; rather, they are part of the foundation on which the theory rests. (Man does not prove the falling of an apple by citing the theory of gravitation. Rather, he proves the theory by citing the fall.)

Broad Applicability of Eifo-Slowing

Eifo-slowing applies not just to clocks and periodically cycling objects but also to linear motions, chemical processes, and all other kinds of activities. Clearly if the wheels of a freight train rotate eifo-slowly, the train as a whole must proceed eifo-slowly, and the passengers must talk, read, etc., eifo-slowly.

Rest Length, Rest mass-- or Basic Length, Basic mass? Which Terms are Most Suitable?

"Rest" is a dull, colorless word, and it makes a poor adjective, "Rest length", for example, is an awkward expression (how can a length rest?).

"Basic length" has much appeal, having a functional connotation. The basic length of a spaceship, for example, is truly basic in several respects. It is the greatest length found from any frame. Also, it is the length found from the only frame with respect to which (1) the spaceship itself and the objects therein have familiar shape--wheels are circular, ball-bearings and balloons are spherical, and (2) objects within the spaceship retain their shapes irrespective of changes in orientation. Overall, basic length is the length found from the frame favored by Occam and his razor: the frame that minimizes complications, maximizes understanding.

Accordingly I sometimes use "basic length", "basic mass" instead of "rest length", "rest mass."

"Time Dilation", a Poor Term.

I do not like this widely-used, poorly defined, mystical-sounding term. It has about the same meaning as the ill-advised notion: "his clock runs slow".

Chapter 9: Gamma: Definition and Application

Definition

The quantity γ , called gamma, is defined as

$$\frac{1}{\sqrt{1-(v/c)^2}}$$

where v is the speed of one given frame relative to another given frame.

General Field of Application

Gamma plays an important role in problems that involve (1) two frames that are at high speed relative to one another and (2) the existence, in at least one of the frames, of physical objects that have stable properties, for example length, time-rate, mass.

However, gamma is <u>not pertinent</u> to problems involving pairs of events--unless, judged from one of the frames, the events are cotimey or coplacey (that is, unless one frame is the nulling frame for the given event-pair); in such case, the nulling frame's non-zero delta (Δt or Δx) is equivalent to a proper time duration or proper length.

Use

Gamma's use is as factor in converting from values found from one frame to values found from the other frame. It is a "transframe conversion factor".

Typically, one merely multiplies or divides by gamma --simple!

Range of Values

The result gamma = 2 applies when the relative speed of the two frames is 0.87 ft/ns, or 0.87 c. For lower or higher relative speeds, gamma would be lower or higher. Values of gamma range from 1 to infinity. The highest measured value ever achieved in the laboratory is probably the value reached by electrons in orbit in the greatest synchrotrons -- the value being of the order of 100,000.

The following table may be helpful.

v/c	gamma
0.0	1
0.1	1.005
0.2	1.02
0.3	1.05
0.4	1.09
0.5	1.15
0.6	1.25
0.7	1.40
0.8	1.67
0.866	2
0.9	2.3
0.95	3.05
0.99	7.1
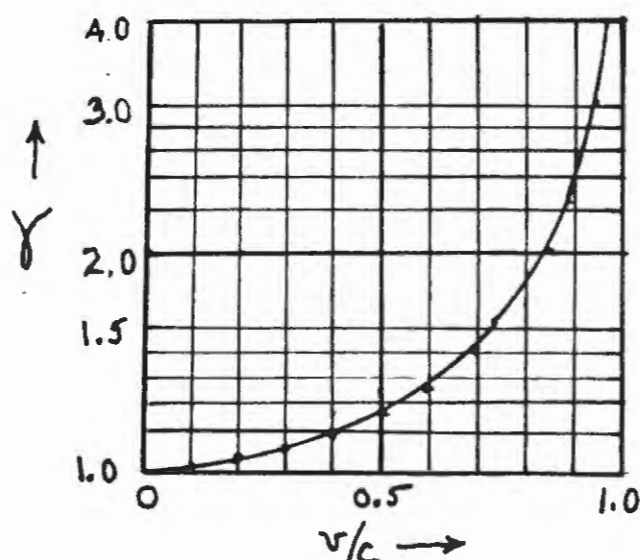
Quick Way to Estimate Gamma in One's Head

If the relative speed is low (less than 0.6 c), one can make a rough estimate of gamma almost instantly in one's head. Take half of $(v/c)^2$ and add 1. The result somewhat exceeds gamma, but not by much.

Examples: Suppose v/c = 0.1. Then $(1/2)(v/c)^2 + 1 =$ 1.005. The true gamma is 1.00505.

Suppose v/c = 0.5. Then $(1/2)(v/c)^2 + 1 = 1.125$. The true value of gamma is 1.15.

Warning: if v/c exceeds 0.5, the method works poorly.

In Many Problems, Values Found Are "Eifo"

If Bakership has a rest-length (proper length) of 100 ft., and Ableship is traveling at speed 0.87c (implying gamma 2) relative to Bakership, then the 50-ft. value Ablecrew finds is an eifo-length -- effective with respect to Ableframe only.

Often, the value initially given is a rest-length or rest-rate of a clock (or other rest property), and accordingly the value found from another frame is eifo.

Does Gamma Have Dimensions?

No. It is dimensionless--is a pure number. It is derived from v/c, a ratio--a pure number.

Does One Multiply or Divide by Gamma?

Before answering this question, we must recognize that there are direct-type and inverse-type quantities; for example, length-type and frequency type. (Recall: wavelength of light vs. frequency of light). Before deciding whether to multiply or divide by gamma, one must consider what type of quantity is at issue.

Ablecrew, in seeking the eifo-length value pertinent to Bakership's 100-ft. rest length, divides 100-ft. by gamma.

If Ablecrew is seeking the eifo-rate of Bakership clocks, Ablecrew divides by 2.

If Ablecrew is seeking the eifo-time-duration value pertinent to Capt. Baker's 30-year proper age, Ablecrew multiplies by 2.

Whenever direct quantities of space are involved, one uses the opposite strategy from when direct quantities of time are involved. (Why opposite? Because of the minus sign in the L-interval.) And vice versa.

Mnemonic Aid

Many of the conclusions or rules, presented above can be remembered by memorizing the term "Dihilear', a sort of acronym for "Divide his length-value and rate-value." If the other fellow's ship has a proper length of 100 ft., this is "his length". So: divide it by gamma to find the results your metersticks and synced clocks will show. Likewise if he finds his pulse rate is 70 per min., divide by gamma to find the rate your tools will show.

Applications to mass, momentum, energy, etc., are discussed in later chapters. Usually, one simply multiplies or divides by gamma.

Chapter 10: Proof of the Outcomes of Interframe Length-Comparisons of Spaceships, or Metersticks, etc.

Introduction

Suppose Ableship and Bakership are passing by one another at 0.87c (corresponding to gamma = 2). Suppose each has a rest-length of 100 ft. When Ablecrew, using its metersticks and synced clocks, measures the length (more exactly, eifo-length) of Bakership, what is the resulting figure?

The answer, 50 ft., is given in a previous chapter. In the following paragraphs I give the proof. It is much like a proof given in Chapter 7, but recast so as to focus on length, not time.

Basic Proof

Let us suppose that the two spaceships are long and slender, and have smooth shiny surfaces, and are passing by each other (at relative speed 0.87 c) so closely that they pass within 10 ft. of one another.

Suppose that a light source on Bakership sends a short pulse of light aimed exactly transverse to Bakership, the timing being such that, by good luck, the pulse hits halfway along Ableship. The pulse is reflected and returns to Bakership and is detected there. Suppose also that, affixed to Ableship, there are two masts that project toward the Bakership line-of-motion and are almost 10 ft. tall and so almost touch Bakership as it passes by. At the top of each mast there is a light-pulse detector and a recording clock. The clocks have been synced in the Ableship frame. The masts happen to be situated far enough apart so that one happens to be very close to Bakership's light source while this source is being pulsed and the other happens to be very close to Bakership's detector while it is detecting the returning pulse. (See Diagram.)

$$x^2 = 10^2 + [(0.87)x]^2$$

solving,

$$x = 20 \text{ ft.}$$
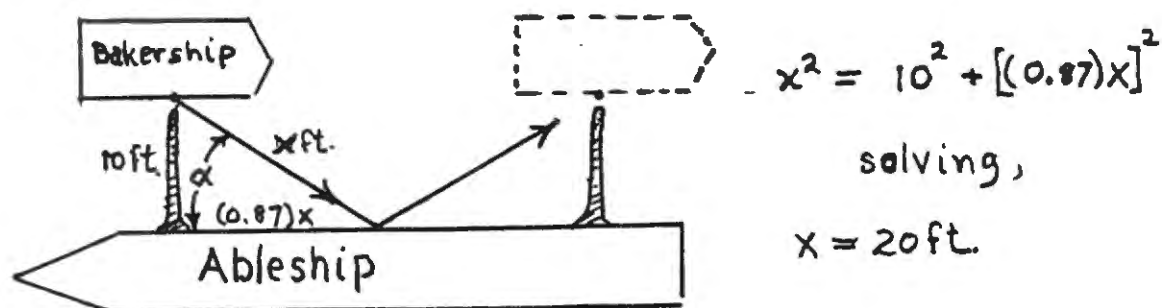
Diagram by Able

According to <u>Bakercrew</u>, the round-trip distance of the pulse is 20 ft. and necessarily the round-trip travel time is 20 ns (since light travels 1 ft. per ns, judged from any frame). In this time interval, Ableship travels, relative to Bakership, (20 ns)(0.87 ft/ns) =17.4 ft. Thus according to Bakercrew data, the effective distance (eifo-distance) D_{Baker} between masts is 17.4 ft.

What does <u>Ablecrew</u> find? As shown in the diagram, it finds the length of each leg of the light-pulse-travel to be 20 ft. Thus it finds the aggregate length of the two legs to be 40 ft., which implies that the per-Ablecrew distance D_{Able} between masts is

$$(\cos \alpha)(40 \text{ ft.}) = (0.87)(40 \text{ ft.}) = 34.8 \text{ ft.}$$

<u>Summary:</u> What is the length D of the between-masts segment of Ableship? 17.4 ft. per Bakercrew, 34.8 ft. per Ablecrew.

Thus the rest-frame (proper frame, Ableframe) value is just twice the value found by Bakercrew.

Or briefly:

$$D_{Able} = 2 \, D_{Baker}$$

General case

For the general case of length D (of a rigid object at rest in Ableframe) as found by Ablecrew and Bakercrew, when the interframe relative speed is v,

$$D \text{ found by Able} = \left(\frac{1}{\sqrt{1 - (v/c)^2}} \right)(D \text{ found by Baker})$$

or briefly: D found by Able = (γ)(D found by Baker)

The result derived above applies not merely to spaceships and other esoteric or mundane objects but also to the measuring tools themselves: metersticks, clocks, etc.

The eifo-shortening applies, of course, only to the component of length parallel to the lines of motion. There is no transverse effect. (See Chap. 11.)

Eifo-shortening does not apply to a pair of events. It applies only to physical objects.

<u>Big Triple-Play Rule: the "SSS Rule".</u>

When a spaceship travels past you at high speed, you find:
the ship's clock is "<u>S</u>low",   ⎫ eifo
the ship itself is   "<u>S</u>hort",   ⎬
the ship itself is   "<u>S</u>chwer" (heavy). ⎭ quantities

Chapter 11
Absence of Transverse Effects

The Question

Special relativity deals with frame-to-frame relationships of events, clocks, metersticks, spaceships, etc., distributed along, or parallel to, the line of relative motion. One of the effects dealt with is "shortening" (or "contracting"), better called eifo-shortening (or eifo-contracting).

But what about eifo-shortening along a transverse line --a line perpendicular to the line of motion? For example, does Ablecrew, speeding past a transversely oriented pole, find the pole to be eifo-shortened? -Or merely eifo-thinned? If there is longitudinal eifo-shortening, is there also transverse eifo-shortening?
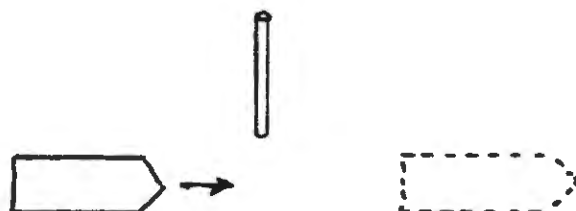
The Answer

The answer is No.

A powerful reason is that there is no logical way of defining the axis toward which the contraction (eifo-contraction) would tend.

Suppose there are twin spaceships traveling along together, side by side, passing by Ableship. Will each of the twin ships transversely contract about its own axis, or will the contraction occur with respect to the system axis--a line midway between the ships? What if one ship has ten times the mass of the other-- will this influence the location of the line the contraction implies? Or suppose one ship is 50 ft. ahead of the other, or 50 miles ahead. Will this influence the location of the implied line?

Consider some basketballs within the spaceships; will each basketball contract toward its centerpoint? Or will each move as a whole toward the centerline of its ship?--Or toward the centerline of the pair of ships?

The fact is, it is impossible to formulate an acceptable physical rule, or basis, for choosing one line rather than another as the axis toward which eifo-contraction would occur. Any rule anyone proposes can easily be shown to be ridiculous. All lines parallel to the lines of relative motion have equal standing. Therefore eifo-contraction cannot occur with respect to any one of them. Therefore there can be no such contraction.

By the same token, there can be no transverse eifo-expansion.

For longitudinal eifo-effects, the situation is very different. The two given frames define a direction-and-counter-direction (loosely called line-of-motion) and the eifo-contraction is along it.)

Concentric-Cylindrical-Pipes Proof

Take two identical 10-ft-long, 3-ft. diameter parallel pipes A and B. Arrange for them to approach each other at high speed along a straight line. Will observers riding along on A find B to be transversely eifo-contracted , and will they see it to be so eifo-slender that it will pass harmlessly through the opening of A? What about the converse case; will observers riding along on B find A eifo-contracted and see it pass harmlessly through the opening of B?

Clearly it is impossible, during the confrontation, for A to pass within B while B is passing within A. The whole proposition makes no sense. Conclusion: Objects at high relative speed do not eifo-contract transversely.

Which is right:
  this?
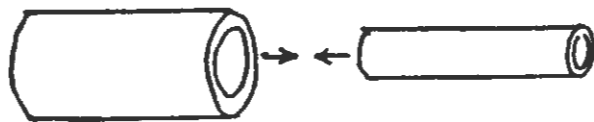
Or this?
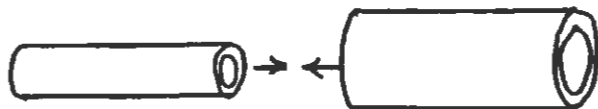
Train-on-Track Proof

Suppose that, every day, a train travels on a long straight track, and suppose there are always many pedestrians standing beside the track, observing train and track. Suppose on a certain day the train goes past at much higher speed. If it eifo-contracts transversely, as judged by the pedestrians, it will be too slender

to match the track and so will slip between the rails -- will be derailed "internally". But passengers on the train would find the set of tracks to be transversely eifo-contracted, causing "external" derailment.

Which diagram
is correct?
(Neither)
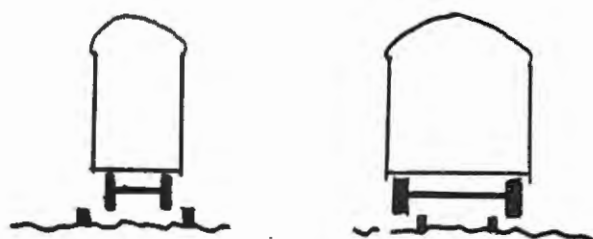
These are contradictory conclusions. So they are invalid. Which means that there is no transverse eifo-contraction.

Note Concerning Transverse Arrays of Clocks

If two spaceships, each wih several synced clocks, are passing by each other, each oriented transverse to the line of motion, each observer team will agree that all clocks in one ship are in sync (read alike) and all clocks in the other ship are in sync (read alike). The two syncs may differ, but there are no variations along either of the transversely oriented ships--no transverse effects.
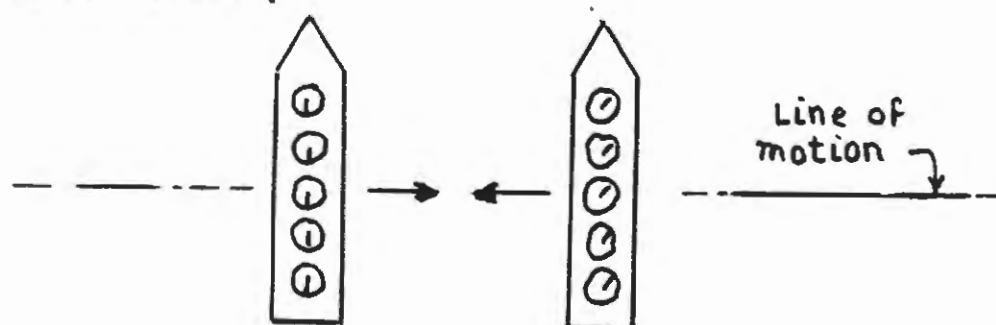
The absence of transverse eifo-contraction applies broadly --applies to spaceships, metersticks, clocks, etc.

It is surprising that most textbooks fail to mention the basic reason for the absence: the impossibility of defining the axis toward which the effects would tend.

Chapter 12: The Lorentz Equations

The famous Lorentz equations (or "Lorentz-Einstein transformations") focus on a single event (not an event pair!), and they relate place-and-time data determined in one frame to the place-and-time data determined in another frame. The following scenario presents the equations and shows how they are used.

Case 1: The Data Concerning Place

Able stands at one end of his straight driveway. This end is the origin of coordinates used by him, and the driveway lies along the positive x axis. He gazes at a pole at the other end of the driveway. He (with his observer team) has found the driveway-length (call it x) to be 87 ft.

The event: At noon, according to Able's extensive set of synced clocks, lightning strikes the pole.

Able says: "I find this event to have the coordinates x = 87 ft., t = 0 nanoseconds."
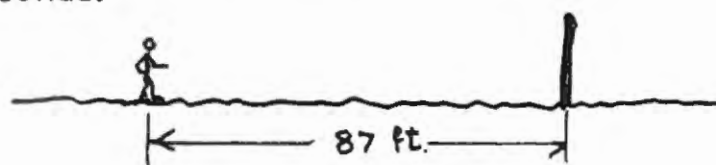
Baker comes coasting past Able and toward the pole. His speed is 0.87 c, which corresponds to gamma = 2. The Bakerframe main axis (call it x'-axis) also lies along the driveway. Baker and his crew have a set of synced clocks.

Baker passes by Able just as Able's wristwatch reads zero (say: t = 0) and Baker's wristwatch reads zero (say: t' = 0).

The Question: What place coordinates does Bakercrew find for the lightning-stroke event?

Answer: x' = 175 ft.

We find this answer by using the Lorentz equation for position:

$$x' = \gamma[x - vt] = 2[87 - (0.87)(0)] = 2[87] \cong 175 \text{ ft.}$$

Note: To represent Able-found space and time values we use simple letters, x, t. For Baker-found values we used primed letters, x', t'. We assume that the Able and Baker metersticks are identical and never undergo change, and we assume that the Able and Baker clocks are identical and never undergo change. Able's clocks are of course synced with respect to Ableframe, Baker's clocks are synced with respect to Bakerframe. We use matched units--foot and nanosecond--which simplifies the equations by eliminating mention of c. We assume that values of y and z, likewise y' and z', are of no interest (or are zero).

Case 2: The Data Concerning Time

For this same event, what time coordinate is found by Baker and his team?

Answer: $t' = -150$ ns.

We find this answer by using the Lorentz equation for time.

$$t' = \gamma[t - vx] = 2[0 - (0.87)(87)] = 2[-75] = -150 \text{ ns.}$$

Discussion:

These two Lorentz equations show how to predict, from place and time values found from one frame, the place and time values found from another frame.

But notice this limitation: The equations do not apply to event pairs and do not provide between-event intervals Δt and Δx. They apply to one event measured from two frames. The equations have nothing directly to do with the L-interval and thus do not play the major role in spacetime geometry. They are less important than L-interval.

How are the equations derived? They follow directly from the key facts and formulas presented in earlier chapters. Standard textbooks on special relativity show the derivations.

Chapter 13
Spacetime Path (Worldline)

Introduction

Most books on special relativity introduce the concept of "worldline" of an object and explain its various uses.

I prefer the name "spacetime path" or "spacetime line". A later paragraph explains why. Often, the shorter word "path" suffices. Or the word "line."

In dealing with spacetime paths, we focus -- not on events--but on an enduring physical object. The spacetime path (1) specifies the successive positions of the object throughout a long (or short) period of time, (2) helps us follow the object's changes in speed and direction, and (3) sets the stage for calculations of the object's cumulative proper time (t_{cp}).

The object may be a proton, a person, a rocket-ship, or any other distinct object.

Drawing the Spacetime Path

An object's spacetime path (line) may be an actual line drawn on graph paper, or a line specified in mathematical form, or it may exist merely as an idea in someone's mind. We shall usually deal with the line as drawn on graph paper.

In its full general form, the path is specified by four variables: t, x, y, and z --that is, a time parameter and the usual three place (space) parameters, such as "east-and-west", "north-and-south", and "up-and-down". Thus the graph is a four-dimensional one --easy for mathematicians to deal with but impossible to draw on a sheet of paper (and also impossible to portray by a solid 3-D model).

An easy simplification is to assume that all of the motions occur along the east-west axis (x-axis). Thus the y and z coordinates can be dropped from consideration. We are left with a plot of time vs. x. Specifically, a line drawn on a sheet of paper.

In our graph of time vs. x, we choose to plot <u>time</u> vertically and <u>place</u> horizontally. It might seem more traditional to plot time horizontally, but for a number of reasons we prefer to plot it vertically. With respect to general relativity, it is more convenient to have time "increasing upward". Also, by plotting time vertically, the horizontal axis (x-axis) is available for indicating "motion-to-the-east."

In drawing the graph's grid (time axis, place axis, and the markings thereon), we use time and place units that are matched. We use nanosecond (ns, 10^{-9} sec.) and foot (ft.) --because light travels almost exactly one foot in one nanosecond. (We could equally well use "meter-of-distance" and "meter-of-time", except that these terms are cumbersome and lack simple abbreviations.)

A given graph always presupposes a given free-float (non-accelerated, inertial) reference frame. The spacetime line describes the changing place of the given object relative to this frame. Regardless of how fast the object moves, or in which direction along the x-axis it moves, always the motions are relative to this one frame. (Curiously, one well-known text states that the line is "frame-independent", has an existence "irrespective of any and all frames".)

Examples of Spacetime Paths

Let me use my laboratory as the reference frame and use my desk and clock as defining the origin, or zero-point, of the graph. Then the position of my file cabinet, 3 ft. east of my desk, has the spacetime path A shown in the following graph. The line is vertical because the file cabinet's position never changes (its x-value stays the same, at 3 ft. east) but the hands of my clock continue to go around and around.
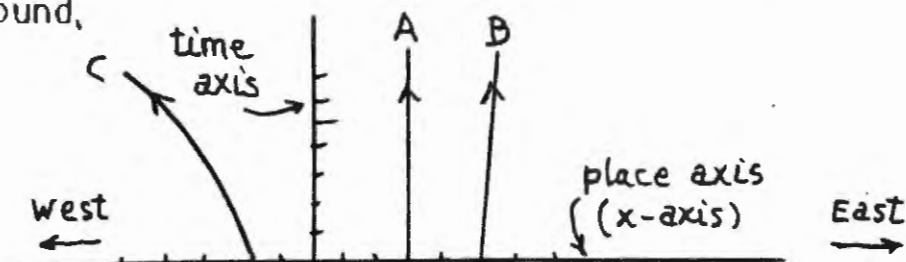
Spacetime path B is the path (again with respect to my desk and clock) of a housefly that is initially 5 ft. east of my desk and is flying steadily east. (The slant of the line is exaggerated, for clarity. In fact the line would be so close to vertical that it could not be distinguished from vertical.)

Spacetime path C applies to a spaceship that, from a standing start just west of my desk, accelerates to higher and higher speed westward. The line tends upward and to the left because the motion is westward, that is, along the negative branch of the x-axis. The line curves ever more strongly away from the vertical because the speed is ever increasing.

Many spacetime paths that are of any interest are curved, because many objects that are of interest are changing speed.

Viewing any spacetime path, we shift our gaze higher and higher along the path to learn about the object's motion at later and later times, that is, as time progresses.

For practical reasons, the spacetime paths we draw are of limited extent. We may not care about an object's motion long ago, or its motion in the very distant future. We draw a line pertinent only to that portion of the object's history that interests us. For a high-speed proton a time period of a microsecond may suffice. For a distant star a period of many centuries may be appropriate. For a traveler to the moon, a week may suffice. Of course, we change the scale of the graph to match the time period chosen.

Spacetime Path of a Pulse of Light

The spacetime path of a directed pulse of light (group of light waves forming a slender ray) is especially interesting. Consider a pulse that is directed eastward or westward along the x-axis. Because the speed is constant, the spacetime path is a straight line. Because the speed is 1 ft per ns, that is, "one-to-one", the line makes an angle of 45 deg. with the x-axis, being plus 45 deg. if the pulse is eastbound and minus 45 deg. if it is westbound.

Spacetime paths of four light pulses
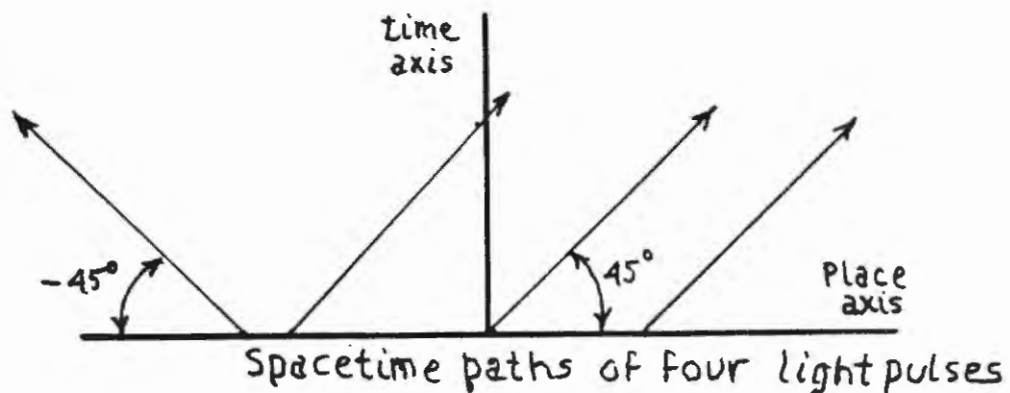
Spacetime Paths of Typical High-Speed Objects

All tangible objects travel at speeds less than c, and accordingly their paths must be steeper than 45 deg., that is, more nearly vertical. The following graph shows the paths of many objects that initially are situated at various locations along the x-axis.

Spacetime Paths of Colliding Objects

It is instructive to draw--on a single graph --the paths of two objects that are traveling along the x-axis and collide. In the following graph, Curves M and N show the collision between a tennis ball and a basketball. Notice that the tennis ball undergoes tremendous change in speed, and its direction along the x-axis reverses, whereas the much heavier basketball undergoes only a small change in speed. If accurately drawn, the changes implied by the graph would be in exact accord with the pertinent conservation laws of dynamics.

Curves P and Q show the collision of two protons that approach my laboratory from opposite directions and with equal speed. Because the speeds are so close to the speed of light, the curve directions are close to the limiting (45 deg.) directions,
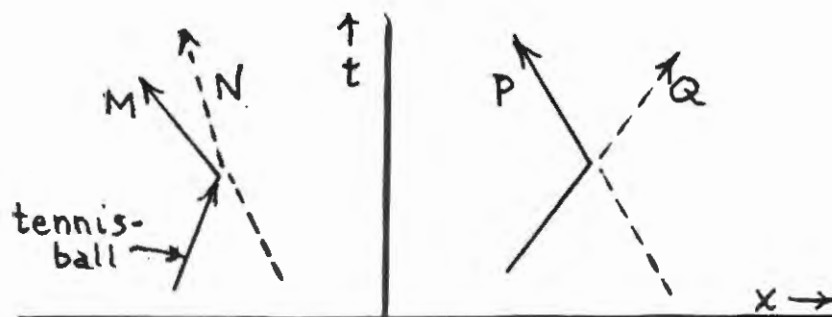
Why "Spacetime Path" Is a Better Term Than "Worldline"

"Spacetime path", defined and explained above, has little or nothing to do with the world --Mother Earth. It has much to do with spacetime. Also, "spacetime path" is a dynamic term, consistent with "path"; a path, by definition, is what something travels along. Contrariwise, "line" is a static term.

Thus "spacetime path" is an entirely appropriate term. "Worldline" is not.

Spacetime Influence Cones

One's understanding of the 45-degree spacetime paths of pulses of light leads to the philosophically interesting concept of spacetime influence cones (called by most writers "light cones"). There are two types of cone: one involves the future (relative to, say, Ableframe's "here and now"), the other involves the past. They are explained separately below. We assume initially that the motions are along the x-axis only, so that y and z do not enter and the diagrams are easy to draw.

Future-Influence Cones

Consider Point P in the following spacepath diagram, and consider Able who is at the origin of the diagram. Is it possible for Able to influence an object or event whose time and place (relative to Able's present frame) are indicated by Point P? Yes. In principle, Able can travel to P and influence an event there. Or, more simply, he can dispatch a bullet or electron or light pulse or some other "agent" that can influence an event there. In summary, Able and P can be "connected" by cause-and-effect.
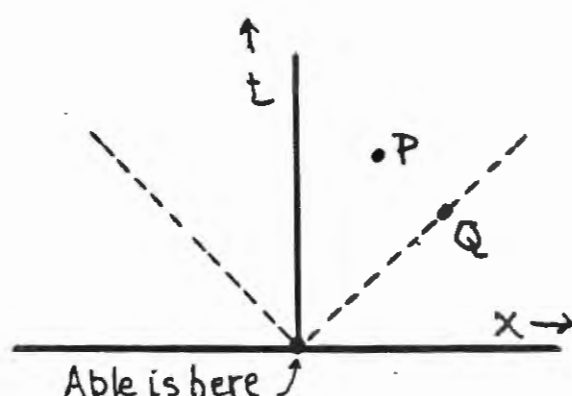
What about Point Q? It lies directly on a 45-degree line. Can Able influence an event here? Yes, but only by dispatch of an agent traveling at speed c--which means dispatching a pulse of light or equivalent.

Summary: Any event specified by a point situated in the cone (actually, a wedge-shaped area) between the two upward 45-degree lines can be influenced by Able by dispatch of a suitable agent (tangible object or light pulse). Any event specified by a point lying on one of these lines can be influenced by a light pulse only. Any event to the right or left of the cone cannot be influenced by Able-- there is "too much distance, too little time."

Past-Influence Cones

Consider Point S in the following diagram. Can a past event, or person, at S influence -- or have influenced-- Able at the present time? Yes indeed--because the place interval (in ft.) is less than the time interval (in nanoseconds). But a past event at Point T cannot influence "Able here-and-now"--because the distance exceeds the available time interval. No agent dispatched at T --not even a pulse of light--can have reached Able now.
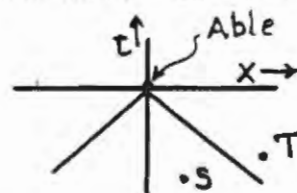

Warning: Always, it is possible that an agent dispatched from T will reach Able at some future time--depending on Able's future travels. At some future time, a new influence-cone will apply to Able, and T may lie within it, as suggested by the following sketch.
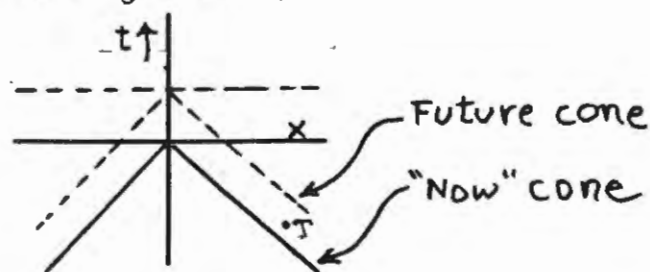
Case Involving x and y

Let us now consider the case where we have to deal with the X- and Y-axes as well as with time. (We still ignore z.) Consider Point P situated in the upper cone as indicated in the following perspective drawing. Here the limits (conical surfaces) are defined --not by two lines at 45 degrees -- but by cones that have a 45-deg. half-angle. Point P lies within the upper cone (future-influence cone), and accordingly Able can influence an event at the time and place defined by P. But consider Point Q which lies outside this cone: an event here cannot be influenced by Able.
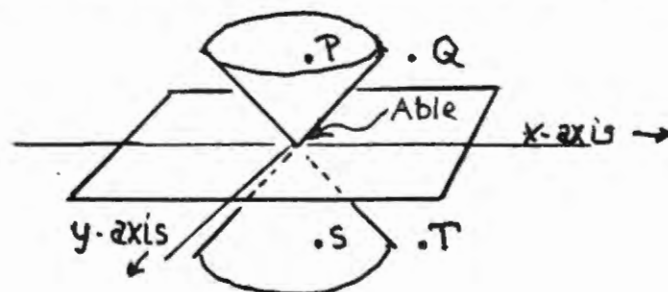
Consider Point S in the <u>lower</u> cone (past-influence cone). A person here can have dispatched a bullet or other tangible object or a light pulse that can influence Able now. But a person at Point T, outside this cone, cannot have influenced Able now: the distance exceeds the available time.

<u>General rule:</u>

> Able, using a suitable agent, can influence any event specified by a point within, or in the surface of, the upper cone.
> Any event specified by a point within, or in the surface of, the lower cone can have influenced Able now-- if a suitable agent was dispatched.

> Points lying in the cone <u>surface</u> qualify only if the agent dispatched is something traveling at the speed of light.

> Regardless of the type, direction, and speed of agent, no event outside the pair of cones can have any causal connection (any influence-connection) with Able-now. There is simply too much distance, too little time.

Computing Cumulative Proper Time (t_{cp}) from a Given Straight Segment of Spacetime Path

Consider the straight spacetime path shown in the following diagram, drawn with respect to Earthframe. The segment implies constant-speed eastward travel of 8.7 light-years in a travel-time of 10 years (all relative to earth). What is the cumulative proper time (t_{cp}) of the traveler --that is, what will his wristwatch show the travel-duration to have been?

Answer: It is the diftor $\sqrt{(10)^2 - (8.7)^2}$ = 5 years.

8.7 light-year

Diftor of this is *5 years*

10 yr
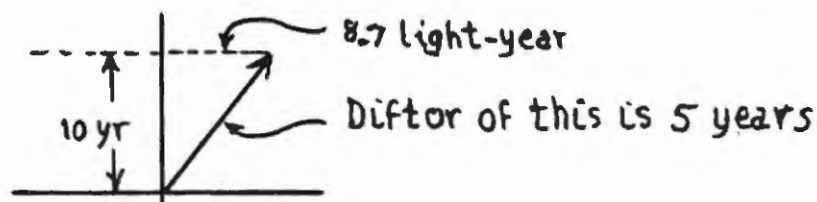
The rule is: first find what Δt and Δx the segment implies, then find the diftor of these.

What is the t_{CP} implied by the following diagram?
Here the Δt and Δx are equal, so the diftor = 0.
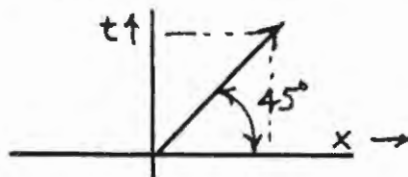
The closer a line-segment's angle-from-vertical is to
45 deg., the closer the diftor -- and the t_{CP} -- comes to
zero.

What About Two Contiguous (Sequential) Segments?

Merely find the t_{CP} for each, and add these. t_{CP} is a
"proper" concept; also it is basic --same value findable (by
appropriate calculation) from any and all inertial frames.
Thus the values are to be added.
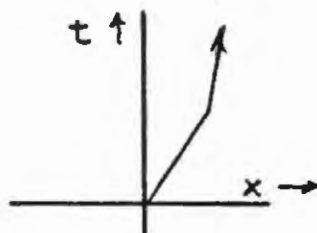
What about a Curved Line?

Here we must divide the line into segments so short
that each is practically straight. Then proceed as above:
add all the small component t_{CP}'s.

In some cases, instead of adding lots of little
quantities, one can, of course, employ calculus--employ
integration.
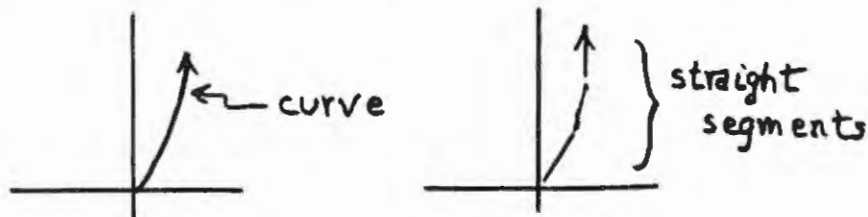
Example of Extensive and Definitive Use of Spacetime Paths

See Chapter 20 dealing with the twin paradox.

Limitations and Awkward Features of the Spacetime-Path Concept

The concept does not apply to events in general -- only to events at which one given object is present, that is, events experienced by one given "traveler".

It is awkward that: the slower the travel, the steeper the line. In most kinds of graphs involving speed, greater speed implies a steeper line.

Also, it is awkward that the line indicative of the greatest possible speed is neither vertical nor horizontal--but at plus or minus 45 degrees.

Chapter 14: Adding Speeds

Introduction:

Increments of speed do not add. If a spaceship speeding past me at 0.5 c launches a tiny rocket in forward direction at speed 0.5 c relative to the spaceship, the speed of the tiny rocket relative to me is not 1.0 c. To merely add 0.5 c to 0.5 c is not permissible. It gives an answer that is too great.

Likewise if Smith approaches a great array of parallel moving sidewalks, each moving 5 mph faster than the preceding one (as judged from the preceding one), and he steps from one sidewalk to the next, his speed relative to me does not change in 5-mph increments, but only in increments that are slightly less. No matter how many times he progresses from one sidewalk to the next-faster one, his speed will never reach c.

Here's another kind of scenario in which simple addition of speeds gives the wrong result: Able speeds eastward past me at 0.9 c and Baker speeds <u>westward</u> past me at 0.9 c. Is Able's speed relative to Baker 1.8 c? No indeed. It is slightly less than c.

Rule for Combining Speeds

If a spaceship has speed v_1 relative to me and launches, in forward direction, a tiny rocket that has speed v_2 relative to the spaceship, the tiny rocket will have, relative to me, a speed of:

$$ v = \frac{v_1 + v_2}{1 + v_1 v_2} \qquad (\text{taking } c = 1) $$

Derivation of Rule

One way to derive the rule is by trial and error. Find the simplest formula that implies:

> if v_1 and v_2 are very small, the combined speed is almost exactly the simple sum of these,
>
> If v_1 and v_2 become ever larger, the combined speed approaches c but can never exceed it.

It is soon found that the above-presented formula is the simplest one that satisfies these conditions.

A formal method of deriving the formula is presented in standard textbooks on special relativity.

Chapter 15

Cause of the Frame-to-Frame Variation in Length Measurement

The Question:

What is the __basic__ cause of the different values of length, clock-rate, etc., found from different frames?

A simple answer is: "Differences in sync." But this answer is not basic.

The basic answer is: "The linkage of time and space."

But what, exactly, is this linkage? What is the effect on, say, measurement of length?

The Answers

What exactly is the linkage?

It is such that the speed of light is found to be the same for all observers. (See Law 2 in Chapter 2.)

What is the linkage's effect on measurement of length?

Observers in different frames, when measuring a given object's "length", arrive at different values.

See proof in Chapter 10. The proof employs the diagram reproduced below. Bakership, traveling at 0.87 c relative to Ableship, produces a light pulse that makes a round-trip to Ableship and back. Bakercrew finds the round-trip to take 20 ns and finds the length of the between-masts portion of Ableship to be 17.4 ft. Able crew finds the round-trip to take 40 ns and finds the length of the between-masts portion to be 34.8 ft. The two results differ by the factor 2, which is the gamma value pertinent to the given speed, 0.87c. See Chapter 10 for details of proof.

Bakership

10 ft.

α

(0.87)x

x

Ableship
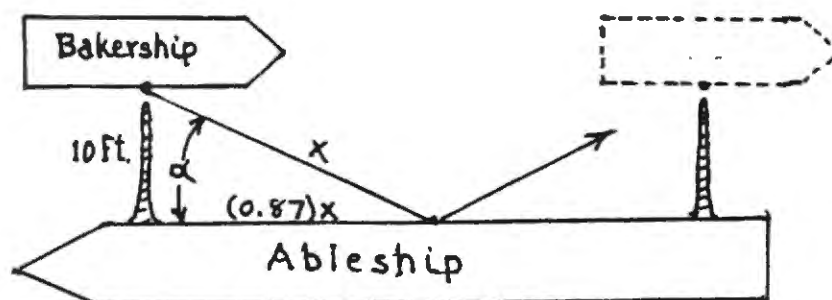
$$x^2 = 10^2 + [(0.87)x]^2$$

Solving,

$$x = 20 \text{ ft.}$$

The line of reasoning presented above is, ultimately, equivalent to a line-of-reasoning based on the frame-to-frame invariance of the L-interval--itself a kind of definition of the linkage of time and space.

<div align="center">***</div>

Similar lines of reasoning apply to measurement of clock-rate. Here too the strange effects observed stem from the linkage of time and space. See Chapter 8.

Query Concerning Shortening

Someone might say: "I have read that the true cause of the small value length (length pertinent to Ableship) found by Bakership is the shortening, or contracting, process caused by high speed. Ableship is at high speed relative to Bakership, and accordingly Ableship has been shortened."

This is not correct. There is no shortening process. What is found by Bakercrew is _eifo_-shortness. Furthermore, eifo-shortness is a state, not a process. There is no process; no physical change in Ableship, no cause of change.

This general subject is discussed at length in Chapter 16.

It is unfortunate that most textbooks imply-- or leave the reader free to assume -- that relative high speed actually shortens an object. Pertinent quotations from textbooks are included in Chapter 16.

Chapter 16: Various Textbooks' Wrong Statements as to "Clock-Slowing" and "Shortening" and Proof They Are Wrong.

Introduction

A survey of 15 textbooks on special relativity reveals many faulty statements--statements concerning the interpretation of various key mathematical relationships. (I am indebted to J. C. Gray for allowing me to borrow his library of books on special relativity.)

Some of the statements are wrong. Most are vague, perhaps deliberately worded so the writer will not have to really face the issues and resolve them. (It is this writer's belief that several authors have chosen to rely on the mathematical equations and have not taken the time to find clear and correct verbal interpretations.)

Many of the statements, even if correct, are worded in such peculiar manner that many students are likely to arrive at false notions and latch onto them permanently.

Examples of Bad Statements

M. D. Mermin: "Space and Time in Special Relativity", pages 37 - 55: "...a light-beam clock runs slower when it is moving than when it stands still." "We are forced to conclude that ... (a moving) clock ...slows down." "The fact that moving clocks run slower than stationary ones..." "That is, when moving, it runs at a slower rate." (The slowing of a moving clock) "...has been observed and quantitatively verified." "We are forced to conclude that moving meter sticks are shorter than stationary ones."

Albert Einstein, "Relativity: the Special and the General Theory", Crown Publishers, Inc. 1961 (translated). p 35: "The rigid rod is thus shorter when in motion with the velocity v relative to... (other body)." p. 37: "As a consequence of its motion the clock goes more slowly than when at rest." (This book was written for beginners and has been translated from German. Surely Einstein knew better!)

M. D. Mermin, Space and Time in Special Relativity", McGraw Hill, 1968, p. 55: "We are forced to conclude that moving meter sticks are shorter than stationary ones." p. 38: "We are forced to conclude that (a moving) ..clock....slows down."

M. G. Moore: "Insight into Relativity", Carlton Press, 1988, p. 26: "...the traveler, or any object moving with him, must contract in the direction of motion." p. 24: "...the traveling clock must run more slowly than the stationary one."

Nevanlinna book "Space Time and Relativity", Addison-Wesley, 1968, p. 131: "...a clock that moves (with respect to ...) is retarded."

J. Schwartz, "Relativity in Illustrations", Dover Publ., 1962. p. 112: "...moving rulers of every kind shrink". p. 110: "Moving clocks of every description run slow."

Shadowitz book "Special Relativity", Saunders Co., 1968. p. 55: "Is the Lorentz-Fitzgerald contraction a real effect or is it merely some trick of measurement? .,.. it is clear that the effect is a real one."

Hans Reichenbach, "From Copernicus to Einstein", Dover Press. 1927 and 1970. p. 68: "...movement exerts a retarding influence on clocks."

J. L. Synge, "Talking About Relativity", American Elsevier, 1970. p. 166: "How can a moving body contract? Because there are no rigid bodies."

P. W. Bridgman, "A Sophisticate's Primer of Relativity", Harper Co, 1962. p. 97: "Relativity theory works in practice, and its metersticks are doubtless shortened when set into motion." p. 101: "...can be accounted for in terms of the change of rate of the transported clock........ the moving clock runs slow..."

Seven Ways of Proving "Slowing" and "Shortening" to Be False.

Way #1: Consider Ableship and Bakership and the clocks therein, and suppose that the ships are at high speed relative to one another. If relative high speed slows a clock, how can anyone claim that Baker's clocks are slow compared to Able's --since this implies that Able's are fast compared to Baker's? To say one clock is slow compared to another is to imply there is asymmetry; yet when two ships pass by each other, the situation is symmetric. (Fact: High relative speed does not make a clock run slow.)

Way #2: The Baker clocks, and Bakercrew also, may be totally unaware of the existence of Ableship and its clocks. Perhaps Ablecrew has taken pains to transmit no signal, no information, to Bakership. How then can the Baker clocks be aware of, or influenced by, Ableship? (Fact: The existence of Ableship has no effect whatsoever on the Baker clocks.)

Way #3: There might be a dozen spaceships passing by Bakership at a dozen different relative speeds. How would the Bakership clock know to which ship--which relative speed-- to respond? Or can it respond to a dozen different extents simultaneously? (Fact: It responds to no ship, no relative speed.)

Way #4: The alleged "slowing" is claimed to be independent of the distance between the spaceships. Suppose the ships pass by each other at a distance of 1 foot or 1,000,000,000 miles. How can the slowing of the Baker clocks be unaffected by this range of distances? (Fact: The Baker clock does not respond to any ship, no matter what its distance.)

Way #5: Suppose the two ships are far apart and the sun is situated between them. Does the high speed of one ship relative to the other have full effect on the other's clocks despite the sun's interposition? Do the ship-vs-ship influences travel freely through the sun? (Fact: There are no such influences.)

Way *6: Suppose that, while Ableship is at high speed relative to Bakership, Bakership is parked on earth. Then if Ableship's relative speed were to make the Baker clocks run slow, it would make all the clocks on Earth run slow. Also, it would slow the rotation of Earth itself (the rotating Earth is a kind of clock!) and slow the revolutions of the planets about the sun. (Fact: Ableship's high relative speed has no effect on clocks, planets, or anything else, whether large or small.)

Way *7: If Bakership's clocks' high relative speed were to slow the rotation of Earth, then the same effect would be produced even by one small wristwatch hurtling through space at that same relative speed. (Earth cannot distinguish between clocks in spaceships and a small wristwatch traveling "naked".) Also the same effect would be produced by a wristwatch that is broken. (Earth cannot distinguish between a watch that is running and one that is broken.) The same effect would be produced by a dust particle at high relative speed. (Earth cannot distinguish a broken watch from a dust particle.) (Fact: This whole line of discussion is ridiculous.)

What about "shortening"? The same seven lines of argument presented above can be used to prove that "shortening" is a false notion.

Reminder: Special relativity relates measurements made in different frames --measurements, numbers. Any writer who introduces the words "slow" or "short" (or implies processes such as "being slowed" or "being shortened"), with no warning, is going beyond the facts and is likely to mislead his readers.

Chapter 17: Dynamics of a Single Object

Introduction

Here we move onward to a new domain of special relativity: the basic kinematic properties of a single object: its mass, momentum, energy, momenergy vector.

Heretofore we dealt with spacetime geometry, or metrics. The main focus was on events and on "where?" and "when?" and on the intervals Δt and Δx. Our concern was geometry -- not mainstream physics.

This has been a bloodless pursuit. Never has attention been focused on a given object per se -- its mass, weight, momentum, energy, etc. Interest was confined to 4-D geometry.

***

Here we start a series of chapters dealing with the physics of actual physical objects --single objects, groups of objects, objects that are colliding with one another. We deal also with light and other forms of electromagnetic radiation, and we deal with the interaction of such radiation with objects -- interactions that can lead to annihilation of objects and creation of objects.

The striking fact is that special relativity plays an enormous role in the effect of relative speed of the measured properties of real physical objects. When an object is coasting past an observer at a speed close to that of light, the effects are large. Strange relationships are encountered. But, thanks to special relativity, they are easily understood.

Relativity plays big roles in considerations of individual objects and also in systems of objects--systems of all kinds. In this chapter we deal just with a single object. The fun comes when we deal with systems.

In this chapter we deal just with a single massive object, for example, a stone, a spaceship, a planet, an electron or other particle.

We inquire how the object's properties, as measured in a given frame, depend on the object's speed relative to that frame.

Discussions are facilitated by use of appropriate words. Several key words are in routine use in many textbooks. Several other terms are introduced here -- to fill gaps and make the discussions shorter and clearer.

Rest-Mass and Eifo- Mass

Before Einstein's work, physicists assumed that a stone, for example, had a fixed mass. They assumed the mass to be the same whether the stone was fixed in the laboratory or coasting past at high relative speed. The mass was usually called "m", and it was regarded as a constant.

Einstein's discovery that time and space are linked upset this simple notion. The discovery implied that the higher the speed of the stone through the laboratory, the greater the mass as indicated by laboratory tools (metersticks, synced clocks, etc.).

"Rest-mass" was the name given to the mass as measured when the object is fixed in the laboratory -- fixed with respect to the mass-measuring tools.

"Eifo-mass" is the name used in this book to indicate mass as measured by tools in a specified frame that the object is passing through. Measurements from different frames lead to different values of eifo-mass. Each value is effective (valid) for the indicated frame and sync only.

Note: We could use the term "dynamic mass", but this might suggest to unwary persons that a given object has two distinct and real dynamic masses. Of course, such is not the case.

Textbooks have no term for "eifo-mass".

Finding Eifo-Mass

Consider a steel ball that, at rest in Bakership, has a mass of 1 kg. (rest mass, or proper mass, of 1 kg as determined with the aid of the Bakership tools). Suppose that the ball (and Bakership as a whole) coasts past Ableship at 0.87c, for which gamma = 2. Suppose the ships pass by one another so closely, with the ball so exposed, that Able is able to exert a transverse (upward, say) force on it and accelerate it in the upward direction --as by use of magnetic forces, or pressure exerted by a laser-beam.

Force
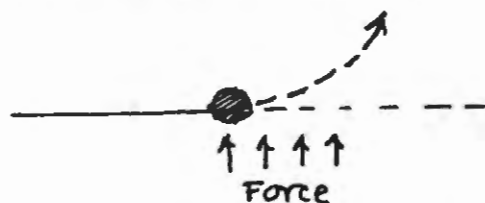
Diagam drawn by Able

Able will find that the acceleration is only half what might be naively expected. The old formula: $F = mA$, or $A = F/m$, is not obeyed.

It turns out that the valid formula is: $A = F/(\gamma m)$.
Thus the mass-with-respect-to-transverse-acceleration, or "eifo-mass," as determined by Abletools, is γm.
Of course, Bakercrew may be unaware that high speed is involved. As far as the crew is concerned, the mass is unchanged--is still the rest-mass. The eifo-mass indicated by Abletools is valid for the Ableship frame but for no other frame.

Why does the term gamma enter? Because the formula for acceleration involves time, and accordingly the period of the Bakership clock's seconds hand is found by Abletools to be different from that of the Ableship clock's seconds hand. Abletools show the eifo-period, or eifo-duration, of the Bakership second to be gamma times that of the Ableship second. (see Chapter 7).
For this reason the speed-increase produced per Ableship second of "pushing" likewise is small; the effect is such as would be expected (by an uneducated person) if the ball's mass really were extra large.

Summary: If m = rest mass and m_{eifo} = eifo-mass, then: $m_{eifo} = (\gamma)(m)$.

Warning: If I try to accelerate the stone along its line of motion, I find the effective mass to be:
(gamma cubed)(m), or $\gamma^3 m$.
This quantity can be called the "eifo-longitudinal-mass."

Momentum Vector

The momentum vector of a steel ball, for example, is defined by its vector direction and vector magnitude--all with respect to, say, the Ableship frame.
The vector direction is the direction noted by Ablecrew. In a simple case, the direction may be along the x-axis in positive sense. In a more complicated situation there may be x, y, and z components which, when added

vectorially in the usual manner (add the arrows, tip to tail), yield a resultant direction: the vector direction.

The vector <u>magnitude</u> is the magnitude of the vector formed by adding the x, y, and z component vectors. It consists of the sumtor of the x, y, and z magnitudes. For example, if these are 2, 3, and 4, the <u>sumtor</u> is:

$$\sqrt{(2^2 + 3^2 + 4^2)} \quad = \text{ about } 5.4.$$

Symbols used:
Momentum = $\vec{P}$ (a vector)
Momentum magnitude: $|\vec{P}| = \gamma mv$ (a scalar)
x, y, and z vector components
of momentum: $\vec{P}_x, \vec{P}_y, \vec{P}_z.$ (vectors)
Magnitude of x , y , and z components:
$p_x, \quad p_y, \quad p_z.$ (scalars).

Kinetic Energy

This quantity is a scalar. The kinetic energy of a steel ball, traveling through my lab at high speed, is found by my tools to be greater than the classically computed value.

I find the value to be: $m(\gamma - 1)$. The value may also be stated as: (Magnitude of total energy – rest mass), or (E – m).

Energy

Until Einstein's discovery, an object at rest in a laboratory was commonly said to have little or no energy (unless especially hot, or compressed, or magnetized, etc.). If the object was in motion relative to the lab, focus was usually on energy of motion --kinetic energy, calculated by means of the formula $(1/2)mv^2$.

But Einstein's work revolutionized the subject. His discovery that time and space are linked implied that energy and mass likewise are linked. More specifically, energy and mass are equivalent--in principle, at least.

In some instances the principle can be converted into dramatic reality. When a uranium atomic bomb explodes, there is a small decrease in rest-mass of matter (of the order of 1% decrease) and an increase in energy in the form of heat, shock-waves, light, etc. (But to convert the mass of, say, a large stone into heat, shock-wave, light, etc. is practically impossible--given the limitations of today's technology.)

The equivalence is specified by: $E = mc^2$.

Because c^2 is such a large number, a small decrease in rest-mass (expressed in kilograms) corresponds to an earth-shaking amount of heat, shockwaves, etc.

In summary, it is now recognized that the energy of an object must be expressed as the sum: rest-mass + kinetic energy.

What about heat energy? Energy of compression, magnetization, etc.? These usually do not require special attention inasmuch as, ordinarily, "rest-mass" is defined so as to include the mass attributable to the heat, compression, etc. Thus it suffices to combine just two terms: rest-mass and kinetic energy.

What is the unit of energy? Because energy and mass are equivalent, we may use either the usual unit of energy (joule) or the usual unit of mass (kg). Usually the latter is preferred.

Energy Vector

For reasons explained below, it is often convenient to regard energy as a vector, defined as the vector whose magnitude is the energy of the given object (or set of objects) and whose direction is normal to (perpendicular to) the x, y, and z axes. We assume a 4-D space and assume each of the four axes (energy, x, y, z) is perpendicular to the other three. (Hard to visualize! But easy to deal with mathematically.)

If the motion is parallel to the x-axis, we can neglect the y and z axes and draw a simple 2-D diagram. It is then conventional to consider the x-axis as horizontal and the energy axis as vertical.

Example: A 1-kg stone is moving horizontally east through the lab and has an energy (rest-mass + kinetic energy) of 1.5 kg, relative to the lab. The situation is diagramed thus:

Energy vector
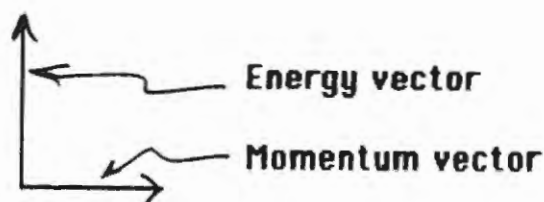
Momentum vector

Momenergy Guide Vector

This is the vector that is obtained by ordinary vector addition of the momentum vector and the energy vector.

Momenergy guide vector

As indicated below, the importance of this vector resides in its direction only.

The vector, fully written out as having four components, is often called a _four-vector_.
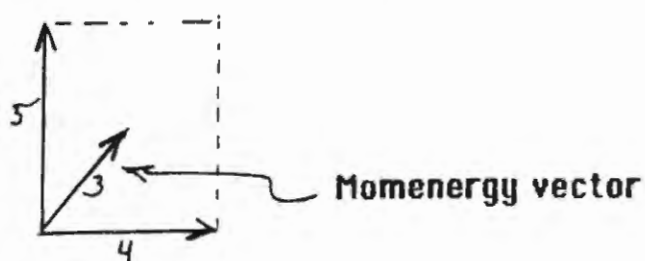
Momenergy Vector

This is defined as the vector having (1) direction same as that of the momenergy guide vector and (2) magnitude that is the diftor of energy magnitude and momentum magnitude.

$$\begin{pmatrix} \text{Momenergy} \\ \text{magnitude} \end{pmatrix} = \sqrt{\begin{pmatrix} \text{energy} \\ \text{magnitude} \end{pmatrix}^2 - \begin{pmatrix} \text{momentum} \\ \text{magnitude} \end{pmatrix}^2}$$

Example: Consider a stone that is coasting east (parallel to the x-axis) through my lab at high speed and has energy magnitude 5 kg and momentum magnitude 4 kg. Then-- measured by tools fixed in the lab--the momenergy magnitude is:

$$\sqrt{5^2 - 4^2} = \sqrt{9} = 3 \text{ kg}.$$

The following diagrams summarize the facts:

Momenergy vector

Five Uses of the Momenergy Guide Vector and Momenergy:

Momenergy guide vector and momenergy are outstandingly important concepts in relativistic dynamics. If you know these vectors of a given object (relative to your frame) you can at once find the following properties of the object (relative to your frame):

Its momentum. This is the momentum component of the guide vector.

Its travel-direction in 3-D space. This is the direction of the momentum component of the guide vector.

Its energy magnitude. This is the magnitude of the vertical component of the guide vector.

Its rest-mass. This is the magnitude of the momenergy vector, found by diftor process.

Its kinetic energy. This is the excess of the energy magnitude over the rest-mass.

Warning: Some writers combine the concepts of (a) momenergy guide vector and (b) momenergy vector. They employ a single term and put it to double use. This can be confusing.

Frame-Invariance of Momenergy Magnitude

With respect to a given object, measurements made from different frames lead to the same value of momenergy magnitude. That is, momenergy magnitude is frame-invariant.

Is this surprising? Hardly! As indicated above, the quantity is simply the rest-mass, and --by definition -- rest-mass is the same for all frames.

The frame-invariance can be very helpful to persons trying to solve problems involving two or more frames. Momenergy serves as a kind of Rosetta stone to interface the findings from different frames.

What Else Is Frame-Invariant?

Not direction. Not momentum. Not energy. Not kinetic energy. Not momenergy.

Momenergy magnitude , being frame-invariant, is unique.

Properties of a Photon

A photon cannot pause, cannot rest. It has no rest-mass.

Therefore its <u>magnitude of energy vector</u> must be equal to its <u>magnitude of momentum vector.</u> Only if these two quantities are equal is the diftor zero (implying: rest-mass zero).

The following diagram applies to a photon that, relative to some frame Ableframe, has an energy of 2 electron volts (ev). Notice that the energy and momentum vectors are of equal length.
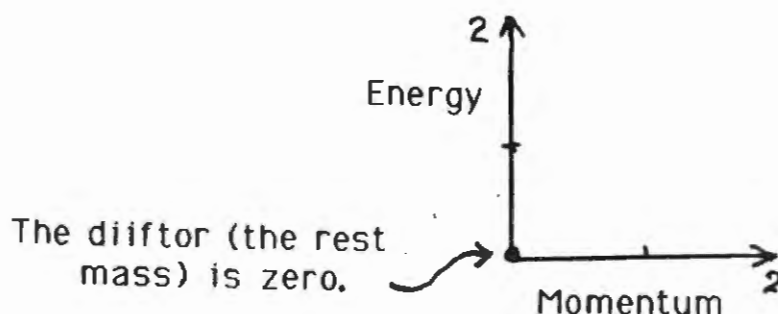
Relative to some other frame, both vectors might have, say, magnitude 5. Whatever the frame, both vectors have the same magnitude and therefore the diftor (the rest mass) is zero.

What Do L-Interval and Momenergy Magnitude Have in Common?

Both are frame-invariant. L-interval is of paramount importance in spacetime geometrics. Momenergy magnitude is paramount in spacetime dynamics.

Both may be computed by diftor process.

Chapter 18: Dynamics of a Group of Objects

Introduction

Here interesting questions arise. What is the overall result when, say, a group of five stones coasts past the lab? What is the result if the stones, moving at slightly different speeds, collide and bounce off one another? Or if five hunks of putty coast past, some of them colliding and clinging together?

What are the pertinent physical rules? What remains unaffected -- remains same before and after?--Is <u>conserved</u>?

What property, if any, is found to be the same with respect to all frames?--Is <u>frame-invariant</u>?

Suppose the given system includes some photons? What then?

The good news is that the concepts of <u>momenergy guide vector</u> and <u>momenergy</u> again provide easy answers.

Key Quantities

With respect to a given inertial frame:

- The momentum vector of a given group of objects is simply the sum of the momentum vectors of the individual objects.
- The group's energy vector is the sum of the energy vectors of the individual objects.
- The group's momenergy guide vector is the vector sum of the group's momentum vector and energy vector.
- The direction of the group's momenergy vector is the same as that of the momenergy guide vector.
- The magnitude of the group's momenergy vector is the diftor of the group's energy vector's magnitude and the momenergy guide vector's magnitude--and is the rest mass of the system.

What Is Collision-Constant?

What remains constant ("conserved") as various collisions occur among the various given objects? All of the above-mentioned vectors of the group, magnitudes of the group.

Notice that all this is true whether the collisions among the objects are elastic or non-elastic. The outcomes, expressed as the group's vectors and magnitudes, is the same for stones, rubber balls, and hunks of putty.

Example: Suppose two glass marbles, traveling along the x-axis of my lab, collide elastically. Suppose that before the collision, one is traveling east with energy magnitude 5 and momentum magnitude 3, and suppose the other is traveling west with energy magnitude 2 and momentum magnitude −1. How do we specify the key properties of the group before the collision?

Answer: Add the energy magnitudes to obtain 5 + 2 = 7. Add the momentum magnitudes to obtain 3 − 1 = 2. Then the momenergy guide vector has the vertical and horizontal components 7 and 2 respectively. Its direction is arc tan (7/2) or about 74 degrees. The momenergy magnitude (rest mass), found by diftor process, is:

$$\sqrt{7^2 - 2^2} = 6.7.$$
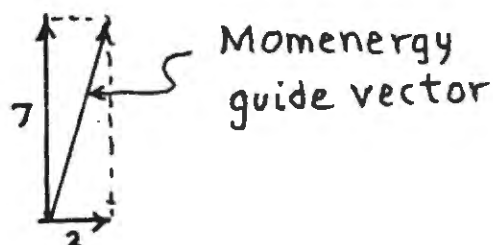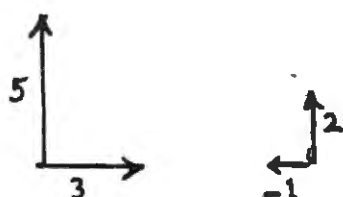
What are the properties of the group after the elastic collision? Same!

A related example: As above, except that the colliding particles are of putty and, on colliding, stick together. What are the properties of the group after the collision?

Answer: Same as above.

How can this be? If objects collide and stick together, they are somewhat heated. But if the collisions are elastic, there is little or no heating. Must not the overall momentum differ in these two cases--and the overall energy also? No. If the objects stick together, there is "more heat energy, less kinetic energy", and the two effects cancel. The overall

energy is the same. The overall momentum also is the same; likewise the momenergy guide vector, likewise momenergy vector and momenergy magnitude (rest mass). (Note: when an object is heated, it has more energy, therefore more mass.)

Still another example

Suppose two steel blocks are clamped together when there is a compressed helical string between them, and suppose that this system is at rest in my lab. Then with respect to my lab the system has a certain momentum (zero!), momenergy guide vector, and momenergy.

How are these vectors affected if the clamps slip and the spring hurls the blocks away from one another at high speed?

None! Same rules apply. System vectors unchanged.

General Case

Given any set of objects, or objects and photons, an observer-team may add their momentum vectors, also add their total energy vectors, and so obtain the system momentum vector and system total energy vector. These two system-vectors, by vector addition, determine the momenergy guide vector; and the diftor of the magnitudes of the two system-vectors is the momenergy magnitude, that is, the rest mass of the system.

Any other observer team will obtain the same rest mass of the system.

Note: It may be that most, or all, of the objects are at high speed relative to the observer-team, and all have much momentum and little rest mass. Nevertheless the system momentum may be small or zero and the system rest mass may be great.

Limitation pertinent to photons: A set consisting solely of photons (having, say, various directions and energies) should not be regarded as a system. However, if the system includes some matter, and the photons will be absorbed by the matter, the photons should be considered part of the system and account should be taken of the momentum and energy of each.

Disintegration and creation of particles The rules presented above apply straightforwardly to events involving disintegration or creation of particles. The rules are put to routine use in analyses of high-energy collisions of electrons, positrons, protons, antiprotons, etc.

Warning Regarding "Sum of Properties" vs. "Property of the Sum"

In dealing with momenergy magnitude, it is essential to distinguish "sum of properties" from "property of sums", more exactly, distinguish these two quantities:

(1) the sum of properties of a given set of objects before they interact with one another, that is, before we are required to consider them as a system;

(2) the properties of the system as a whole after interaction has occurred (or even before interaction--if we already have assurance that there will be interaction).

Consider, for example two counter-traveling photons. Each individually has zero momenergy magnitude (zero mass). Therefore before they interact the sum of momenergy magnitudes is zero; two times 0 is 0. But when the photons are considered as part of a system (are capable of interacting), the system's momenergy vector has a greater-than-zero magnitude and has a direction.

Two-Sentence Summary of Rules of Relativistic Dynamics of a System of Tangible Objects and Photons

With respect to a given reference frame, the mass (rest mass) of such a system is the diftor of (a) the magnitude of the sum of the energies of the items and (b) the magnitude of the vector sum of the momenta of the items.

The obtained value of mass is the same (1) whether, among the items, there are collisions, (2) whether any collisons are elastic or inelastic, (3) whether the evaluation is made before or after any given collision, and (4) irrespective of choice of reference frame.

Chapter 19:
Eleven Short Paradoxes

Introduction

Special relativity offers an abundance of intriguing paradoxes. "Such-and-such must be true -- yet it can't be true!" Such is the feeling of newcomers to the field. The fun comes in trying to reconcilé the opposing views.

Always they can be reconciled. Usually the reconciliation follows from focusing on synchronization of clocks--focusing on the fact that each frame has its own synchronization, its own criterion of "a single instant pertinent to two different locations".

The underlying train of argument starts with the experimental fact that the speed of light is the same judged from any frame, which implies that events cotimey in one frame are distimey in all other frames, which in turn implies that no two frames have the same synchronization.

Below are presented eleven short paradoxes, some of which are found in many textbooks. Others are new. Some are new and very difficult.

The world-famous twin paradox is taken up in following chapters.

Warning: Strictly speaking, what we are dealing with here are not paradoxes at all. They are simply facts -- straightforward logical facts. To experts, they are straightforward. But to most non-experts they seem very strange: they seem to harbor impossibilities, self-contradictions, and mysteries. For simplicity, let us call them paradoxes.

Barn-and-Pole Paradox

This paradox --a well-known classic--is important in showing that eifo-values can be higly important. They are not broadly respected, but can have "life-and-death" importance to persons in the given frames, given scenarios.

The pole is coasting at high speed (0.87 c, for which gamma = 2) toward a barn that has doors at each end. The pole enters through one doorway and leaves through the other. Judged by Mr. Poleman who is astride the pole, the length of the pole is 100 ft. Judged by Mr. Barnman, who is standing beside the barn, the length of the barn is 75 ft. At exactly noon by Barnman's clocks the pole is wholly within the barn, and Barnman's sons close both doors and, after a few nanoseconds, reopen them. The pole continues onward, emerging from the barn without ever having touched either door.

How can a 100-ft. pole be wholly within a 75-ft. barn with closed doors?
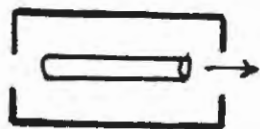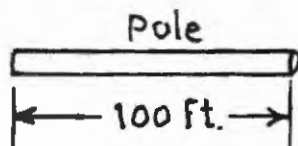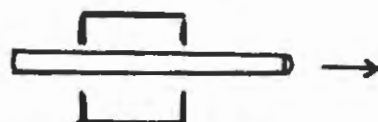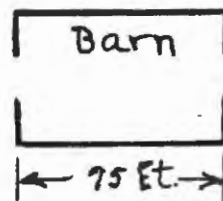
Diagram by Barnman   Diagram by Poleman

Solution: Barnman's account is simple enough; the barn doors were closed at a single instant defined by his clocks (one at each door), clocks that had been synced in his frame. Thus he can say: "At a single instant, both doors were shut and the pole was within the barn."

But judged by Poleman, the two clocks were not synced--and the entrance door was not shut until long after the exit door had been opened--long after the pole had begun to emerge from that door. In other words, judged by Poleman and with respect to his synced clocks, the door operations were conducted sequentially, not

simultaneously.

Thus the paradox is easily explained by anyone aware of the fact that each frame has it own unique sync.

Misguided "clamping" scheme proposed by Shadowitz

The author of a well-known textbook on special relativity, in asserting that the "shortening" of the pole has broad reality, has tried to bolster his claim by stating that, in principle, Barnman could, at a single instant, seize the pole along its entire length by means of a long clamp -- with the result that the pole would become stationary in the barn frame with the "short length" firmly "frozen" for all to see.

Such a scenario, or thought experiment, is untenable, even in principle. Judged from the pole's frame, the clamping would not be instantaneous: the front end would be clamped first, the rear portions later. Consequently the rear portions, before being clamped, would slam into the already-clamped front portions, producing tremendous compression. The compression would be so extreme (would produce such violent interatomic collisions) that the entire pole, and the clamp also, would be converted to an exploding mass of white-hot gas or plasma and an enormous amount of electromagnetic radiation (light, x-rays, etc.). The amount of kinetic energy produced would exceed that from an atomic bomb. In summary, the pole, clamps, and barn would cease to exist.

Shadowitz's thought experiment, far from proving the reality of the pole's "shortness", proves the contrary. The clamping operation, intended to be passive, turns out to be an aggressive and devastating transformation that converts a pole into an exploding plasma. In the attempt to passively "freeze" the pole, all Hell has broken loose. Why? Because of the attempt to convert   an eifo-property into a broadly-respected property.

References: Shadowitz, "Special Relativity", Saunders Co., 1968;   W. A. Shurcliff reports of 5/20/90 and 5/21/91.

Two-Oppositely-Traveling-Trains Paradox

There are two trains, each a mile in proper length, traveling in opposite directions along a single-track railway. Train A is approaching Centerville at 0.87c from the west and Train B is approaching Centerville at 0.87c

from the east. Fortunately, at Centerville the railway is double-tracked for 3/4 mile (see sketch) and the trains pass by one another here without incident.
How can this be?
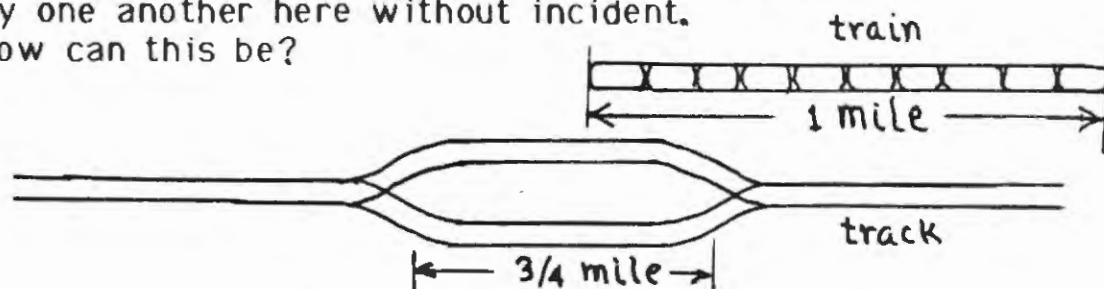
Answer: Judged by persons standing beside the railway, each train has an eifo-length (length effective with respect to Centerville) of only 1/2 mile. Thus the double-tracked stretch is amply long.

The explanation is much the same as that for the barn-and-pole paradox.

Attempt at Rebuttal Someone may say: "Something is wrong here. The people on any one train will find the double-track region to have an eifo-length of only 1/2 of 3/4 mile—length of only 3/8 mile. Clearly, two trains each a mile long cannot pass by each other here. Even if they are judged only a half-mile long, they are too long for a 3/8-mile-long bypass."

The person making this comment has overlooked the fact that people on any one train find the other train to have an eifo-length far less than 1/2 mile and find the other train to be proceeding more eifo-slowly than would be naively calculated.

The simplest way to analyze the problem is to consider the facts as judged by person in the Centerville frame.

Four-Train Paradox

On a broad expanse of flat land in Kansas there are four railroad tracks (two are east-west, two north-south) that cross in such a way as to define a square 2/3 mile on a side—all as judged by track maintenance men standing nearby. Four trains are running on the tracks —one on each track. Each is traveling at 0.87 c relative to the maintenance men. (Note: gamma = 2.) Each train has a rest-length (proper length, length as determined by persons on the train) of one mile. At noon according to the maintenance men all four trains are passing through the crucial region—traveling along track segments defining the square. Yet no train touches any other train; there are no collisions; all four trains pass by uneventfully.

Can four one-mile-long trains be accommodated on a track array that forms a 2/3-mile square?
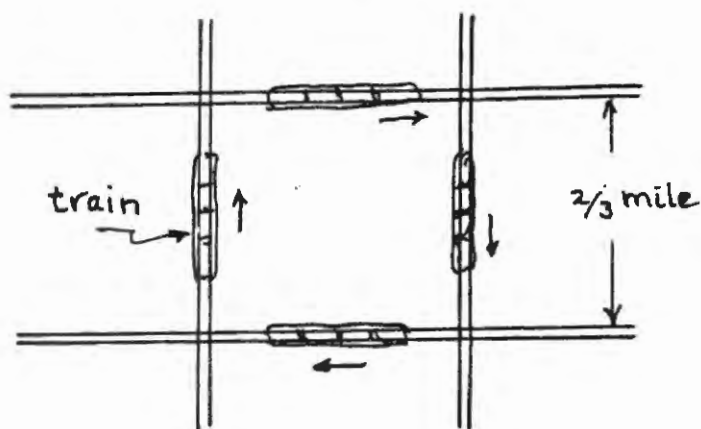
Solution: The answer is yes. With respect to the frame of the maintenance men and their definition of one instant, each train's two ends are, at one instant, only 1/2 mile apart, that is, one mile divided by gamma. Each train's eifo-length is 1/2 mile. So there is no problem.

Attempt at rebuttal: Smith says: "Something smells! An observer A seated near the center of a given Train A finds, at a certain instant defined by his synced clocks, that the "square" is "contracted" and the given train A is much longer than the contracted track segment; the given train extends across the two transverse tracks.

The same may be said for each of the four Trains A, B, C. D: at the same instant, each extends across two tracks. There is bound to be a terrible wreck."

The rebuttal fails. The four trains constitute four different frames, Thus the four observers' findings cannot be combined. It is true that observer A finds Train A to extend across two tracks at a given instant judged from that train, and a corresponding situation is found from each other train, But the results cannot be combined. Only one frame's observers --the observers on the ground --can deal in simple and decisive manner with all four trains at a single instant, They find no interference.

Moving Sidewalks Paradox

A man in Boston steps onto a moving sidewalk --sidewalk #1 that is moving east at 5 mph relative to ground. From here he steps onto an adjacent parallel sidewalk #2 that is moving east at 5 mph relative to sidewalk 1. From here he steps onto an adjacent parallel sidewalk #3 that is moving east at 5 mph relative to sidewalk #2. And so on, indefinitely.

Yet his speed, relative to ground, never reaches c, the speed of light.

How is this possible? How can an infinite number of transfers to faster sidewalks never produce speed c ?
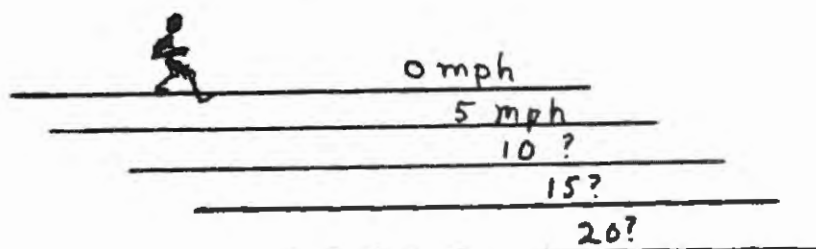
Solution: The speed of each sidewalk is 5 mph faster than the predecessor --relative to the predecessor -- but not relative to the ground. With respect to ground, the speed increment is less. Why is this? Because the predecessor-sidewalk frame has its own sync, and this differs from the ground-frame sync. Differences in sync lead to different values of time intervals, which in turn produce differences in speed values.

For the first hundred or so sidewalks, the differences in sync are trivial, the differences in time-interval values are trivial, and the differences in speed (differences between values computed relative to predecessor frames and values computed relative to ground) are trivial. Thus to a high degree of precision, the first few speeds are 0, 5, 10, 15, 20, etc., mph.

But when attention is turned to sidewalks that, relative to ground, are at, say, 50 or 75% of c, the situation is very different. The speed increments are much smaller than 5 mph, and, for successive sidewalks, the increments approach zero.

To accurately combine the successive speeds, use should be made of the standard formula for addition of speeds. See Chapter 14.

Two-Separating-Spheres Paradox

Able and Baker are coasting close past each other at high speed (say, 0.87c), Able traveling east and Baker west. The passing occurs at exactly noon by Able's clock and Baker's clock. During the passing, the men's fingers touch and a spark is produced and a pulse of light spreads out in all directions. Able finds the spreading wave to be spherical, with himself at the center.

Likewise Baker finds the wave to be spherical with himself at the center. (Each man has a widespread array of metersticks, synced clocks, and light detectors that evaluate the location of the spreading wave.)
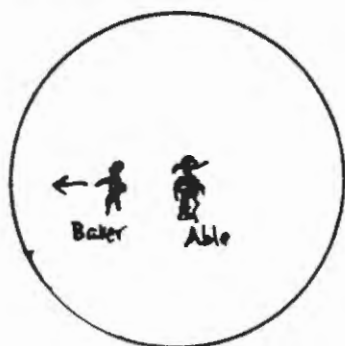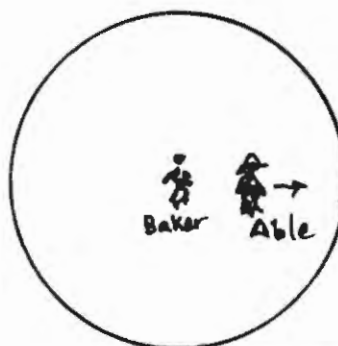
Diagram by Able   Diagram by Baker

As the wave continues to spread, the two men become ever farther apart, yet each continues to find the wave to be spherical with himself at the center.

Is there is a single spherical wavefront? Can each of two far-apart men be at the center?

Solution: Able finds the various portions of the wavefront to be equidistant from him at any given instant--an instant defined by his set of synced clocks. But judged from Baker's frame, Able's measurements were made improperly; in particular, the measurements at the east and west portions of the wavefront were made at different instants and necessarily give a false picture of the location of the wavefront (and location of the center). The opposite view is, of course, held by Able: he asserts that Baker's measurements were made at a variety of instants and give false results.

In summary, there is no contradiction. Each man's conclusion is eifo-valid: valid with respect to his frame. Each man is "eifo-centered" with respect to the spherical wavefront. Neither man's view has any general validity.

Note: As regards the shape of the wave, the two men are in agreement. Both find the shape to be spherical. (Why? Because the speed of light is the same measured from any frame; it is the same for all directions of spread.)

Elliptical Wheel Paradox

Able, standing beside a long straight road, sees Baker riding past on a bicycle at relative speed 0.87 c (for which gamma = 2). Able's special instruments imply that the bicycle wheels are elliptical, with width equal to half the height.

How can a bicycle progress smoothly when its wheels are elliptical?

Baker

Solution: At any one instant judged by Able, the front and back portions of a given wheel have positions that are strangely close together, suggestive of an elliptical shape. But according to the cyclist, and with respect to his definition of "instant", those two positions were determined by Able at different instants and therefore cannot be used in judging the shape of the wheel. The cyclist can correctly say: "The so-called elliptical shape is an artifact of Able's concept of simultaneity. With respect to all other frames (that is, all other definitions of simultaneity) his finding -- 2-to-1 ellipticity--is invalid."

In summary, Able finds the wheels to be eifo-elliptical, Baker finds them to be circular. No contradiction! One quantity is eifo; the other is basic, that is, extreme and broadly respected.

Equidistant Traveler Paradox

Able is to remain on Earth. Baker is to leave Earth and travel at 0.99 c toward a distant star. Charlie is ordered to take off from Earth at the same time Baker does and travel at such speed that he (Charlie) will always find himself equidistant from Able and Baker.

Accordingly, Charlie chooses a speed, relative to Earth, of about 0.495 C. He is roundly criticized for this choice. Why?

Answer: If Charlie is to find himself equidistant from the two men, his speed relative to earth must be 0.868 c. This will also be his speed relative to Baker.
Proof: Using the formula for addition of speeds (addition, in this case, of 0.868 c and 0.868 c) we find that Baker's speed relative to earth is:

$$\frac{(0.868 + 0.868)\ c}{1\ +\ (0.868)^2}\quad =\quad 0.99\ c\ .$$

Oppositely Traveling Speeders Paradox

Relative to Able, two men are departing at high speed in opposite directions. Baker is traveling west at 0.99 c and Charlie is traveling east at 0.99 c. So: how fast is Baker traveling relative to Charlie? At about 1.98 c?

Baker   Able   Charlie

Answer: No. At 0.99995 c.
How can this be?

Solution: Consider the situation as judged from Baker's frame. Baker finds Able to be traveling east at 0.99 c and finds the speed increment of Charlie relative to Able to be very small. Relative to Able the increment was large, but Able's frame's sync is very different from Baker's, and therefore Baker cannot use Able's 0.99 c value directly. Rather, he must use the standard formula for addition of speeds. He finds Charlie's speed to be:

$(0.99c + 0.99c)/(1 + 0.99 \times 0.99) = 1.98c/1.9801 = 0.99995c.$

Reversing Helix Paradox

Take a long round rod, inscribe a straight black line along it, then mount the rod on a horizontal axis.

Some Non-Relativistic Phenomena

Rotate the rod at high rate about its longitudinal axis. Rotate it clockwise as seen by Able who is facing east and is looking east along the rod. To him, the black line appears curved. It has the form of a left helix. (Note: the speed of rotation is assumed to be very high--so high that any actual rod would burst from the great centrifugal force. In this paradox, this practical difficulty is to be ignored.)

Baker, who is facing west and is looking west along the rod, finds the sense of the rotation to be <u>counter</u>clockwise and finds the black line to have the form of a <u>right</u> helix.

All this has nothing to do with relativity, everything to do with the finite speed of light. The light reaching Able from a distant portion of the rod is light that originated earlier than the light from the nearer portions. Such delay, due to the travel-time of light, is the cause of the apparent "rotational lag" of the distant portion, a lag that seemingly converts the black line into a left helix.

Relativistic Phenomena

Suppose Able has a team of thousands of assistants situated at 100-ft. intervals along the rod and all having clocks synchronized in his frame. This team (free of any speed-of-light complication because each man's sight-path is so short) finds the black line to be straight.

But if some observer—Team C is traveling at high speed along the rod, it will find the black line to be eifo-curved -- specifically, eifo-helical. If some Team D is traveling at high speed in the opposite direction, it will find helicity of opposite sense.

Team C result:

Team D result:

If Team C is traveling east, it will find eifo-right-helicity. If Team D is traveling west, it will find eifo-left-helicity. --always assuming that the rod is rotating clockwise as seen by Able, who is looking east.

Summary: High speed can create eifo-helicity in a straight line on a fast-rotating object.

Explanation: The phenomena follow directly from the FF rule ("Front First rule").

Note: If, before the rod is set to rotating, a gently helical (low helicity) line is inscribed on it, an observer team traveling at high speed along the rod will find the helicity of the line increased or decreased, depending on which way the team is traveling. Under some circumstances the observer team will find an eifo-helicity that has reversed sense --right helix converted to eifo-left, or vice versa.

A difficult question: How "tight" an eifo-helix is possible--assuming the black line was inscribed straight? Two facts apply here. The faster the rod is rotating, and the greater its diameter, the tighter the eifo-helix seen by the fast-traveling team. Also, the faster the team is traveling along the rod, the tighter the eifo-helix. What is the tightest possible eifo-helix achievable?   (The author has no answer!)

Bursting Wheel Paradox

Preliminary statement:
        Consider a large-diameter wheel that has a slender rim, and suppose it is situated in my lab. Suppose the wheel rotates faster and faster. Then a given small segment of the rim will be at higher and higher speed relative to my lab, and therefore its "length with respect to my lab" (its eifo-length, judged by me) will be shorter and shorter. This is true of every segment. Thus a person may ask: "Will the wheel break up into many segments, with gaps between them?"

← Will this segment, for example, shorten, leave gap?
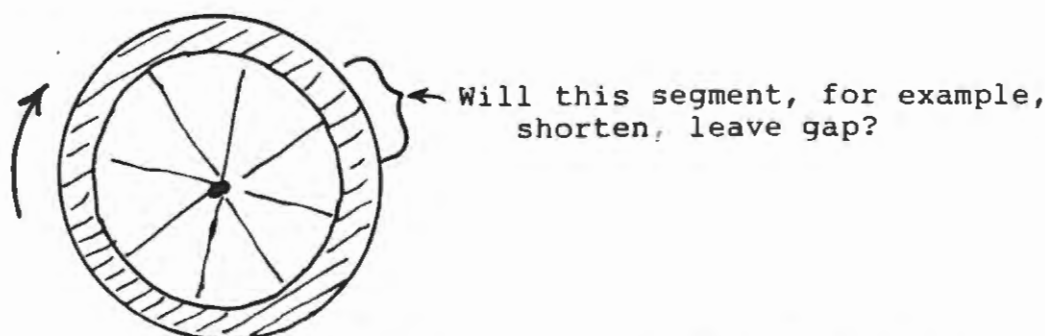
        How is this question to be answered? Will there be a dozen segments and a dozen gaps, or a million segments and a million gaps?
        The paradox is that one can find no basis for deciding on the answer to this question.
        One might propose that the spokes will shorten, so that a smaller perimeter is in order. But the spokes will not shorten; each is moving transverse to its length, and a rule of special relativity is that there are no transverse effects, real or eifo.

        Discussion: Of course, even at modest rotational speeds the wheel will expand somewhat under centrifugal forces. This will occur long before gamma is as large as 0.000,01 --long before the rotation rate reaches values producing significant reductions in "length with respect to my lab". Also, centrifugal forces will cause the wheel to expand and burst long before any signficant relativistic effects arise. Therefore the paradoxical question needs no answer.

Note: When a long straight train is traveling so fast that it has a much reduced "length with respect to my lab", the subject of gaps does not arise. There is plenty of space in front of the train and behind it --to accommodate gaps of any magitude. But the rim of a rotating wheel has no ends; there is no accommodation.

More realistic (and more difficult) form of the paradox:

The Harvard-Mit Cambridge Electron Accelerator, when completed in 1962, was the world's most powerful accelerator of electrons. In the circular orbit ("ring"), 750 ft. in circumference, the electrons achieved an energy of 6 GeV (billion electron volts) and speed 0.999,999,998 c, a speed for which gamma = 12,000.

The orbiting electrons, following a circular path, and accelerated to higher and higher energy, formed a circular array (ring) bearing some resemblance to the rim of a gigantic wheel.

Evacuated steel tube, stationary in lab

Electrons traveling at 0.999,999,998 c
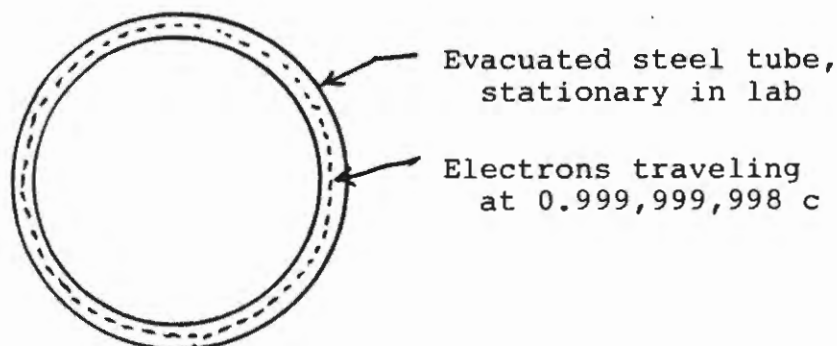
As the electrons were accelerated to higher and higher speed, there was no tendency for the electron ring to "burst" under centrifugal force, because that force was opposed by a strong magnetic field (provided by a series of 18-ft-long magnets) that increased in strength as the electron speed increased.

The revised question: When such a "750-ft.-circumfence ring" of electrons speeds up--resembling the rim of a wheel rotating faster and faster-- does each portion of the ring necessarily become eifo-shorter, judged by the lab team, and does the ring break up into small segments with gaps beween? How many segments? What length of gaps?

Answer: This is a highly technical matter and a full
answer is necessarily long and technical. A brief answer
is that to accelerate the ring of electrons, you must have,
at various locations along the ring, accelerating stations,
called "rf cavities". These accelerate the individual
electrons and at the same time forcibly group them in
bunches and forcibly maintain constant distance between
bunches (distance judged from lab frame). Thus any
tendency toward "shortening" or "producing lengthening
gaps" is forcibly and automatically counteracted.

Thus the paradox, or problem, dissolves into technical
questions as to how the positions of the components of the
ring are artificially controlled.

Approach-to-the-Big-Bang Paradox

Suppose the age of the universe (time since the Big Bang as
found by our astronomers) is 16 gigayears (billion years).
Suppose a local astronomer photographs a galaxy that is found
from red-shift data to be 12 giga-light-years away--the light
received here today started on its journey 12 gigayears ago.
Does the photograph display the galaxy as it appeared at age
4 gigayears? (16 - 12 = 4)

No. Why not? Because of the high-speed-expansion of the
universe and the linkage of time and space.

Consider two events at which the galaxy was present:
Event 1: the Big Bang.   Event 2: a small explosion on the galaxy
when it reached age 2 gigayears. With respect to the galaxy, the
Δt between the two events is 2 gigayears and the Δx between
them is zero (because the galaxy was present at both).

Suppose, finally, that the galaxy has always been in motion
with respect to earth--always at a speed 0.87c, for which
gamma = 2. Then with respect to Earth, aging effects on the
galaxy have been eifo-slow by a factor of 2 compared to those
on earth.   Effective with respect to Earth, the Δt of the two
events is 2(2 gigayears), or 4 gigayears. (Chapter 7 presents
the rules pertinent to eifo-slowing.)

The consequence? Astrophysicists on Earth may summarize
the situation thus: "We note that the galaxy is 12 giga-light-
years from earth, and therefore (a) the light received here now
is light that left the galaxy 12 gigayears ago (judged from
Earth), and (b) when that light was leaving the galaxy, the
galaxy had achieved an age of 4 gigayears judged from Earth but
only 4/2 = 2 gigayears of proper age, that is, age judged from
the galaxy itself.

To the great delight of astrophysicists, the resulting photograph shows the status of the galaxy a mere 2 gigayears after the Big Bang--excitingly close to the Big Bang.

Some galaxies are at even higher speed relative to Earth, and the resulting "bonus" from the relativistic age effect is even more dramatic.

General rule as to the proper age of a galaxy:

Let A be the age of the galaxy at the time (relative to Earth's astronomers' clocks) when the pertinent light was emitted. Let the galaxy's speed relative to Earth be such as to imply a certain gamma (γ).

Then the proper age of the galaxy as portrayed in the photographs is A/γ.

Note: Although the speed of the galaxy plays a big role in the "youthfulness bonus", the direction of the motion plays no role. Even if the galaxy's motion were toward Earth, the bonus would be the same.

Chapter 20   Twin Paradox

Introduction

The most intriguing paradox of special relativity -- one of the world's greatest paradoxes -- is the twin paradox. Featured in nearly all books on special relativity, it has sparked several great debates among scientists and philosophers.

The most famous debate was that between the British scientist Herbert Dingle and the American scientist Edwin McMillan. Over a period of several years the prestigious periodical "Nature" carried their hot letters, each man claiming destruction of the other's arguments. Dingle claimed the paradox was dead wrong. McMillan claimed it was correct. McMillan won.

A <u>brief form</u> of the paradox is: "One twin remains fixed on Earth while the other speeds to a distant star and back--and on arriving back is younger than the stay-at-home twin."

Below, we state the paradox in full detail. Then we prove it to be correct. Several different proofs are given--all essentially equivalent.

The paradox

Two identical 21-year-old brown-haired twins, called Earthtwin and Traveltwin, are living together happily on this Earth. Earthtwin always remains on Earth, but Traveltwin decides to travel at high speed to a certain star and back. The star (at rest with respect to Earth) is 8.7 light-years from Earth, as determined by persons on Earth. Traveltwin's spaceship will travel at the constant speed (relative to Earth and star) of 0.87 c; this speed implies gamma 2, which makes the arithmetic simple. The men have identical wristwatches.

While Earthtwin remains fixed on Earth, Traveltwin takes off for the star-- at noon on 1/1/96, according to   both men's watches. On arriving at the star, Traveltwin immediately starts home, again traveling at 0.87 c. The turn-around requires a short period of enormous acceleration toward Earth.

Earth

Star

Traveltwin

Earthtwin

On Traveltwin's arrival back at Earth, the two men compare results. Earthtwin, referring to his own watch and calender, finds his cumulative proper time (t_{cp})--time between saying "Bon voyage!" and "Welcome home!"--to be 20 years.

Traveltwin's watch and calender show his t_{cp} (his total travel time) to be 10 years.

The men are astonished --and more astonished when they notice that whereas Earthtwin, with his wrinkled skin and white hair, looks old, Traveltwin's skin is still smooth and his hair is still brown. The two men are dumbfounded: "This is impossible!" says Earthtwin. "Totally impossible! " agrees Traveltwin.

But a passing physicist says: "Not at all! Of course Traveltwin is now much younger than Earthtwin. Necessarily so --in view of Traveltwin's prolonged high-speed round-trip!"

Preliminary Remarks Concerning Proof

Proving the validity of the paradox is simple--if the "listener" is already familiar with special relativity.

But to convince a typical layman is difficult, sometimes impossible. Why? Because the basic facts of special relativity are counter-intuitive. They violate common sense. Yet common sense is what the layman relies on. He does not realize that the subject is technical and counter-intuitive, and unless he is willing to learn the pertinent facts of special relativity, he may forever remain unconvinced.

Presented below are several proofs that the twin-paradox outcome is correct. Different proofs may appeal to different readers. Also, the fact that there are several different proofs may help reassure doubtful persons. In a sense, all of the proofs are equivalent; all are based, ultimately, on the linkage of time and space.

Presented first is a solid quantitative proof based on the basic law of the frame-invariance of L-interval. Several short proofs follow.

Proof Employing L-Interval Invariance

Consider the scenario's first two events: Event 1, Traveltwin's departure from Earth; Event 2, his arrival at star. These define the first leg of the trip.

Earth Star

Event 1 (start) Event 2
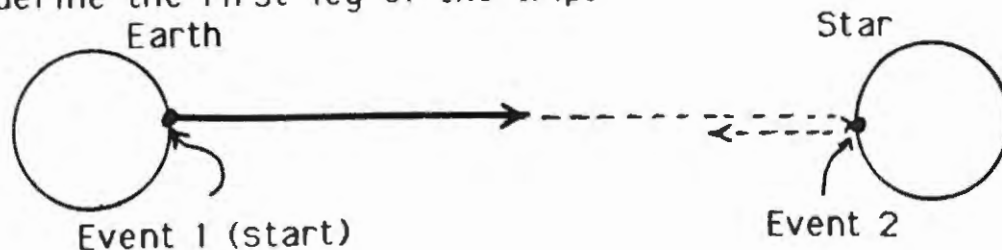
What numbers apply to this leg ? Earthtwin knows that the Earth-to-star distance is 8.7 light-years and Traveltwin's speed relative to Earth-and-star frame is 0.87c (all as specified in the scenario). Therefore Earthtwin concludes that the first leg of the trip took 10 years; Δt = 10 yr. In summary, the Δt and Δx values found from his frame (Earth-and-star frame) are 10 years, 8.7 light-years.

What values does Traveltwin find? His distance value is zero--because he was present at both events. For him, Δx = 0.

Now we invoke the law stating that, for a given pair of events, observers in different frames find the same L-interval, that is, the same diftor of Δt, Δx. Thus the following equation applies. It contains three "knowns" and one "unknown", namely Traveltwin's Δt:

$$\sqrt{(10)^2 - (8.7)^2} = \sqrt{(\Delta t)^2 - (0)^2}$$

or:
$$\sqrt{25} = \sqrt{(\Delta t)^2}$$

or: Traveltwin's Δt = 5 yr.

A similar calculation for the second leg of the trip (return leg) gives a similar value, 5 yr., and accordingly Traveltwin's value for the round-trip is 5 + 5 = 10 yr.

In summary, the first-leg trip time is 10 years according to Earthtwin, 5 years according to Traveltwin. The same values apply to the second leg. Therefore for the round-trip the Earthtwin and Traveltwin t_{cp} values are 20 years and 10 years. Q.E.D.

Qualitative Proof Employing Co=Small Rule

Consider the first two events. They occur far apart relative to Earthtwin but at the same spot relative to Traveltwin (he is present at both events). Thus traveltwin has the smaller Δx--namely zero. Therefore he must have the smaller Δt --according to the Co=Small Rule, explained in Chapter 7 ("He who has smaller Δx has smaller Δt").

The same applies to Events 2 and 3, which pertain to the return trip. Again Traveltwin has the smaller Δx, therefore the smaller Δt.

For the round-trip as a whole, Traveltwin's value of round-trip travel time (his cumulative proper time, t_{cp}) is the sum of two small Δts, and Earthtwin's t_{cp} is the sum of two large Δts. Therefore Earthtwin's watch reads ahead of Traveltwin's, and Earthtwin's body appears more aged than Traveltwin's. Q.E.D.

Can any of the t_{cp}-difference be attributed to the vigorous acceleration Traveltwin experiences on reversing his direction at the star? No. We assume that Traveltwin and his watch are so rugged as to suffer no physical damage, and we note that, in any case, any effect of acceleration would be independent of the Earth-to-star distance, whereas the Δt and Δx values are proportional to that distance. (Questions as to acceleration are discussed in Chapter 22.)

Qualitative Proof Employing Spacetime-Path Diagram

The following spacetime-path diagram, drawn with respect to the scenario's only unchanging frame (Earth-and-star frame), shows the three events and the spacetime paths of the two men.
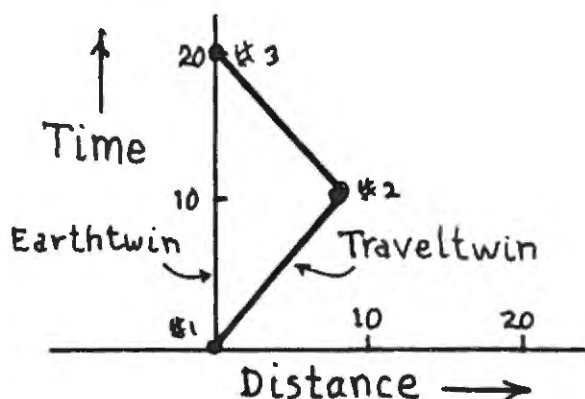
The spacetime path of Earthtwin is vertical whereas the spacetime path of Traveltwin is non-vertical. Therefore Traveltwin's t_{cp} is less than Earthtwin's. All as explained in Chapter 13.

Quantitative Proof Employing "Eifo-Shortening rule"

Because Traveltwin is at high speed relative to Earth-and-star, and because the 8.7 light-year distance was determined by equipment fixed in the Earth-and-star frame, the "eifo-shortening rule" applies to Traveltwin first-leg trip (from Event 1 to Event 2). The rule dictates that the eifo-travel-distance found by Traveltwin is less than the travel-distance found by Earthtwin. The relative speed of the two men is the same judged from either frame. Therefore the trip-t_{cp} found by Traveltwin is necessarily less than that found by Earthtwin. Specifically it is half the value found by Earthtwin, since the 0.87 c speed implies that gamma = 2.

> Note: Use of the "eifo" distance in connection with Traveltwin is permissible because this distance is fully effective with respect to his frame, even 'though not recognized from any other frame. (The term "eifo" is explained in Chapter 8.)

The same argument applies to the second leg of the trip. Accordingly it applies to the entire round-trip. Traveltwin's t_{cp} is half Earthtwin's. Q.E.D.

Experimental Proofs

We can't accelerate human beings to relativistic speed, but producing relativistic-speed muons is easy. These meson particles, with a half-life of the order of a microsecond, are produced in abundance in high-energy physics laboratories. Produced from collisions of high-energy protons or electrons, they are "born relativistic". During experiments in which they are detected at different locations along a 100-ft, or 1000-ft. path, the experimenters routinely find that the measured half-life (eifo-half-life) may be greater than the micro-second value by a factor of 10 or more.

Confirmation of the twin paradox is provided also by comparing two high-precision clocks, one of which remains in the laboratory while the other is transported for several days in a high-speed airplane. The traveling clock has been found to lag behind by a small but detectable amount (of the order of a nanosecond).
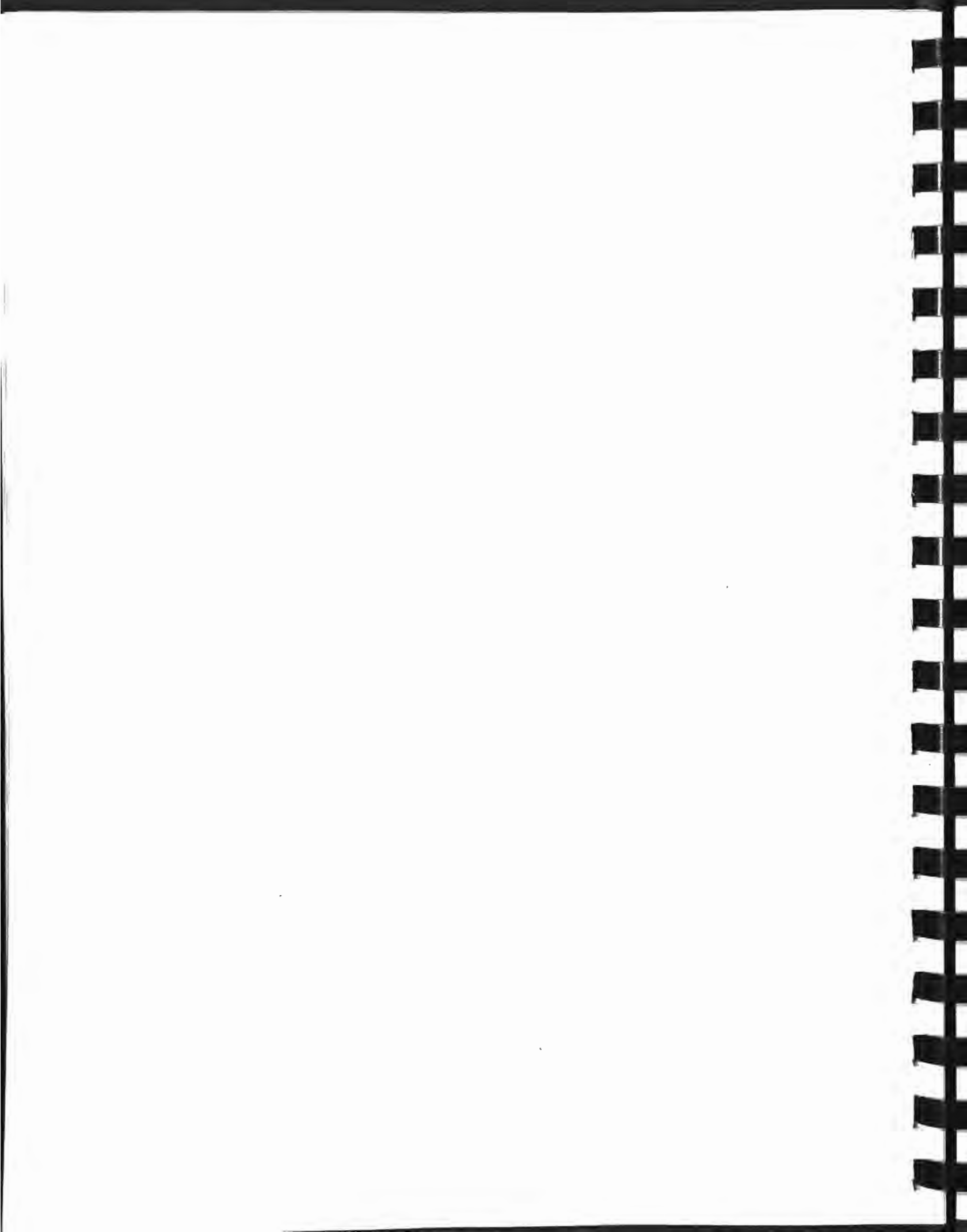
Chapter 21   Attempts to Refute the Twin Paradox

Introduction
One does not fully understand the twin paradox until one has examined various attempted refutations and seen wherein they fail. Countless persons have tried hard to demolish the paradox -- but in vain.

Attempt by White
Mr. White says: "We don't have to think of Earthtwin as stationary and Traveltwin as traveling east. All motions are relative. Therefore with full validity we can think of Earthtwin—and—star as traveling west and Traveltwin as stationary. Thus in all there are two alternative scenarios, and they have equal standing. Therefore either man could be younger than the other."

White's remarks are not significant. He is right, of course, to the extent that, as Traveltwin starts his trip, there are two ways of describing the situation, and they are symmetric. Thus it is true that, at the outset, there is no single "Traveltwin is younger" outcome.

But there is more to the travel than "outset". There are specific events, specific Δt's and Δx's. They must be taken into account before meaningful comparison can be made.

Attempt by Brown
Brown says: "I see how to improve White's thought. Consider the entire first leg of Traveltwin's trip. Throughout that leg the situation is symmetric. So, when Traveltwin reaches the star, we can equally well conclude that he has traveled and is younger or that Earthtwin has traveled and is younger. Two opposite conclusions! –And same for the return trip."

Brown's remarks contain an impermissible (meaningless) word, "when". "When Traveltwin reaches the star" has no meaning. "When" judged from what frame? "Reaching the star" is a definite event, easily observed from all frames. But "when" has no meaning until a particular frame is specified. "When according to one twin's frame" is very different from "when according to the other twin's frame."

Attempt by Green

Mr. Green says: "Even if Traveltwin is younger than Earthtwin at the end of the first leg, the effect is reversed on the (reverse-direction) second leg. The overall effect is zero --the twins end up the same age."

Mr. Green is mistaken. Relativity effects depend on relative speed, not direction. The effects of the two legs of the trip add.

Attempt by Black

Mr. Black says: "Suppose we agree that, during the first part of Traveltwin's trip, Earthtwin finds Traveltwin's clock to become steadily farther behind Earthtwin's. -And vice versa: judged from Traveltwin's frame, Earthtwin's clock becomes steadily more behind. Now suppose that suddenly an intermediate Referee (who manages always to keep equidistant from the two men) issues the radio command "Halt !" and both twins, when they receive the command, change speed and become stationary relative to the Referee's frame. Everything is then symmetric, and the two twins' clocks must read the same. Thus we have contradictory conclusions: Just before the command the twins' clocks read differently, but just after it they read the same--judged by everyone. This is absurd! No enormous readjustment could occur so suddenly. The whole idea of one clock being behind another is wrong!"

Mr. Black is correct in believing that if Referee is always situated symmetrically (judged by Referee) and transmits the command "Halt" symmetrically and the men stop symmetrically with respect to him, their clocks will be found by him to read alike. Will Earthtwin be surprised that they read alike? No, because, judged from his frame, Traveltwin received the command late, and stopped late --just late enough so that the two men's clocks came into agreement. And correspondingly, Traveltwin will not find the situation surprising because he finds that Earthtwin received the command late and stopped late, just late enough so that the clocks came into agreement. Remember: the two twins received the command at the same instant (cotimey) judged by Referee, therefore the command-receipts necessarily were distimey judged from every other frame. Introducing the idea of an intermediate Referee does not simplify the paradox. It complicates it. (Notice also Mr. Black's impermissible use of the word when --he says "When the twins receive the command." Also he uses the impermissible words "just before" and "just after".)

Attempt by Purple

Mr. Purple says: "We can make the situation symmetric--at the start of the paradox, at least-- by assuming that there is a second star and assuming it to be in the same frame with Traveltwin. Then Earthtwin has one star, Traveltwin has another. Everything is symmetric." No twin can be "younger" or "older".

Purple's proposal would indeed make everything symmetric at the outset. But (a) we have yet to reckon with specific events--and with assumption of an additional star there will be additional events to consider, and (2) what Purple is really creating is a second paradox, a paradox in which the roles of the twins are reversed. He contemplates two symmetric paradoxes, therefore two symmetric final outcomes. But neither paradox is explained. Nothing is refuted. Adding a second star has added complexity, solved nothing.

Attempt by Mauve

Mr. Mauve says: "You agree that, at the outset, Earthtwin finds Traveltwin younger and Traveltwin finds Earthtwin younger. These are two "eifo" results. How can -- suddenly -- two eifo-results become real results?"

Mr. Mauve is mistaken. Neither twin can conclude anything about age (t_{cp}) until there is a second event. Once the second event (the arrival at star) has occurred, each twin has a definite Δt (definite t_{cp}). The two values are real. And they differ. The term "eifo" applies to computed properties such as length of a ship, rate of a clock, or mass of a baseball. It does not apply to raw data such as Δt or Δx values.

Attempt by Magenta

"OK, I accept that the clocks will show different readings. Clocks are meant to show time. They keep ticking or clicking or whatever. But people are different. They consist of flesh and blood. They neither know nor care about passage of time. So why should one twin have more wrinkles and white hair than the other?"

Answer: Processes are occurring in clocks, and processes occur in people. The heart, with its steady beat, is a kind of clock. The daily wake-and-sleep pattern is a kind of 24-hour clock. Every part of the body has its processes. Δt and Δx values, and t_{cp} values, apply to the body just as fully as to clocks.

Attempt by Pink

Mr. Pink says: "You have invoked the co=small rule, pointing out that Traveltwin is at every event, so always his Δx is zero--small. But in fact Earthtwin is at Event 1 and Event 3 so his overall Δx also is zero, also is small. So everything is symmetrical!"

Answer: The co=small rule applies to scenarios in which each observer remains in one frame, and accordingly it applies to the first leg of Traveltwin's trip (defined by Events 1 and 2) and also to the second leg (Events 2 and 3), but it does <u>not</u> apply to the two events at which Earthtwin is present (Events 1 and 3) because between these events Traveltwin has accelerated (changed frame).

Chapter 22
Crucial Elements of the
Twin Paradox

Introduction

To become fully comfortable with the twin paradox, one must see clearly what features of the scenario are crucial. One must be able to answer various special questions such as those presented below.

Question 1:   What is the earliest stage, or condition, in a round-trip by Traveltwin that permits this statement: "He is younger than Earthtwin, all observers agree on this, and there is no longer any ambiguity."?

Answer: As soon as some second public event is experienced by Traveltwin--event detectable from all frames. Any interested observer team, having monitored the first two events, can proceed to evaluate between-events Δt and Δx values. Because both events occur at Traveltwin, his Δx value is zero and accordingly the co=small rule (Chapter 7) shows that his Δt value is less than that found from any other frame--in particular, less than the Δt found by Earthtwin.

Until the second event occurs, no meaningful question can be asked. There is no basis for computing Δt and Δx values, and no basis for arriving at a comparison. A person's age-increase is his Δt value; if we have no Δt, we have no age value.

A perfectly good second event is, of course, Traveltwin's arrival at star. But various somewhat-earlier events would suffice. For example, his flicking on and off his spaceship's headlight--or any other act (perhaps even a sneeze!) that is detectable from other frames.

Question 2: Suppose the star were not there; Traveltwin, acting on whim, suddenly accelerates back toward Earth. Is Traveltwin younger than Earthtwin?

Answer: Yes. Sudden acceleration is a public event.

Question 3: Suppose Traveltwin never returns to Earth -- comes to a halt when half-way back. Is he younger than Earthtwin?

Answer: Yes. The second event has already occurred and made the comparison clear. No further event is needed.

Question 4: Suppose Traveltwin, on reaching the star, does not change speed but continues onward, away from Earth, forever. Is he younger than Earthtwin?

Answer: Yes. Because a second event has occurred (Traveltwin reached star), the comparison is clear.

<u>Question 5:</u> Suppose there is no star; Traveltwin simply coasts at high speed farther and farther from Earth. Is he younger than Earthtwin?

    <u>Answer:</u> Until there is a second event, there can be no Δt and Δx values, no comparison.

<u>Question 6:</u> Suppose that shortly after Traveltwin starts his trip, Earthtwin stumbles and falls. At that time, which twin is younger?

    Answer: "At that time" has no meaning. Nevertheless the question is challenging and has an interesting answer. With respect to these two events: <u>Traveltwin's departure</u>, <u>Earthtwin's fall</u>, "Δx is zero" applies to him; therefore the co=small rule applies to him; he is younger than Traveltwin.

    But this becomes irrelevant on Traveltwin's arriving at the star--because interest then again becomes focused on the event-pair <u>departure from Earth</u>, <u>arrival at star</u>, and the co=small rule applies to Traveltwin.

    Summary: Correct answers are always given by the co=small rule. But the rule depends on events, and different choices of events (different scenarios) can lead to different answers.

What Role Is Played by Traveltwin's Acceleration at Star?

    Traveltwin's acceleration-toward-Earth, on reaching star, is essential to the completion of the scenario--essential to reunion of the twins so that their watch-readings can be compared and the men's strikingly different appearances will be apparent.

    Is the acceleration essential to having Traveltwin younger than Earthtwin? No. The existence of a <u>second public event at Traveltwin</u> (for example, Traveltwin' arrival at star) suffices. Even if he never slows down--continues away from Earth forever-- he is younger than Earthtwin. All observers will agree,

    Does the acceleration itself cause "youthfulness"? No. This may be seen at once by comparing a very short trip (producing an age difference of only a few days, say)' with a very long trip (producing an age difference of several years). The extent of youthfulness is proportional to the distance,

Which Is Harder to Accept: Traveltwin's Greater Youthfulness or Earthtwin's Greater Age?

Both are easy to accept.

Traveltwin, on the first leg of his trip, feels himself to be "stationary". He is totally relaxed, and "nothing happens". He has but one watch and nothing to synchronize. The trip-time he reports is utterly reliable. The same applies to the second leg; the acceleration has no effect other than to change his direction. His overall trip-time (t_{cp}) of 10 years is cleanly reliable.

Earthtwin likewise considers himself stationary and relaxed. He measures his t_{cp} by referring to his single watch. His t_{cp} of 20 years is cleanly reliable.

Warning: Each man's t_{cp} is cleanly reliable as far as he himself is concerned. But it does not apply to anyone with a different travel history.

Note: Earthtwin and his teammates fixed in the Earth-and-Star system bear the responsibility for determining (a) the Earth-to-star distance and (b) the traveler's speed. The determinations involve use of two or more clocks and these must be synchronized. (The sync conflicts, of course, with syncs in all other frames.) Thus Earthtwin's frame has the complications of being associated with sync and the measurements of distance and speed. Traveltwin has the complication of being successively in different frames.

What Is the Physical Explanation of the Twins' Age Difference?

There can be no basic explanation. Nothing basic in science can be explained!

If we cannot explain time or space, how could we hope to explain the linkage between them? How could we explain why different travel histories produce different aging?

The best we can do is state the momentous experimental fact that the time-space linkage is such that the L-interval is frame-invariant; that is, the diftor of Δt and Δx between two given events is the same irrespective of the frame from which Δt and Δx are measured. The twin-paradox outcome follows directly from this basic experimental fact.

Attempt at Reconciliation

Can we find some way to reconcile the big distance covered by Traveltwin and the short elapsed time (his small t_{cP}) ?

The round-trip distance is 17.4 light-years, reliably determined by Earthtwin and his teammates who are at rest in the Earth-and-star system. Their result is the proper distance.

Traveltwin's elapsed time is 10 years, reliably determined by Traveltwin by consulting his single watch. This result is his proper time duration.

The speed value computed from these values is

$$\frac{17.4 \text{ light-years}}{10 \text{ years}} = 1.74 \, c$$

This speed (Traveltwin's proper speed -- ratio of two proper quantities) exceeds c -- exceeds 3×10^8 m/sec.

Thus if we accept the concept of proper speed (speed that can exceed c), we can understand how such a great distance can be covered in such a short time.

(The subject of proper speed, and speed exceeding c, is discussed in Chapter 23.)

Does "Clock-Slowing" Play a Role?

Many textbooks imply that Traveltwin's small t_{cP} is to be explained by the phenomenon "speeding clocks run slow". They make the claim that high relative speed affects the operating of clocks and all other processes: slows them down. Clocks run abnormally slow, trains and spaceships are slowed, bodily aging (metabolism) is slowed. High relative speed is the cause, and slowing is the effect. Such is the widely made claim. (Chapter 16 presents examples of such claims).

The claim is entirely false, as explained in Chap. 16. It is not only wrong, but gratuitous. It is gratuitous because (a) cause-and-effect are outside the scope of special relativity, and (b) there is no cause, no effect; space and time are linked in such a way that different travel histories lead to different elapsed times.

Distance-Specifier-Ages-More Rule

Earthtwin and his teammates specify the traveler's starting point and goal; they specify, in other words, the distance. This automatically insures that Earthtwin will age more than the traveler.

Why? Because the specifier necessarily finds big Δx between the two events (starting point and goal) and the traveler necessarily finds zero Δx. Thus the co=small rule insures that the traveler will have the smaller Δt.

In summary: he who specifies the distance ages more than he who travels that distance--which may be called "Distance-specifier-ages-more" rule.

Final Remark about Symmetry

At the beginning of the twin paradox, the two men can be thought of as moving symmetrically away from each other, a fact that tempts persons to suppose that the men will age equally.

But the scenario is fundamentally and necessarily asymmetric. Earthtwin specifies the distance, Traveltwin does not. Therefore the die is cast! On reaching the specified goal (star), Traveltwin is younger than Earthtwin, and all observers agree to this.

There is the further asymmetry that Traveltwin, not Earthtwin, accelerates. This is prerequisite to having the twins come together, not essential to Traveltwin's greater youthfulness.

Amusing Changes in the Scenario

Let us retain the requirements that Traveltwin (1) travel to Star and back, (2) travel at constant speed, and (3) be present at Earth at the times Earthtwin calls "zero" and "20 years".

But suppose we permit Traveltwin to abandon the straight-line route and follow an arbitrarily curved route--longer route. Can he make his t_{CP} smaller than 10 years? Yes. By traveling at higher speed along a longer path he can reduce his t_{CP}. In the extreme case (speed is c), his $t_{CP} = 0$.

Suppose, alternatively, we retain requirements (1) and (3) but allow Traveltwin to make one or more changes in speed. Can he then make his t_{CP} smaller than 10 years? Yes. By traveling to star and back at speed c, then waiting (on Earth) 2.6 years until Earthtwin calls "20 years", the traveler's t_{CP} is 2.6 years.

Chapter 23: Can a Case Be Made for Recognizing a Differently Defined Speed (Proper Speed) and Infinite Proper Speed of Light?

A Disturbing Question

Suppose I drive my car along a stretch of highway defined by "0 mile", "1 mile" markers, and suppose my watch shows that I cover the mile in one minute. Is my speed one mile per minute?

If you say "yes", you are endorsing a differently-defined kind of speed. With respect to all standard physics textbooks, you are committing a blunder.

In standard textbooks, speed is defined differently. With respect to the above scenario, textbooks define the speed of my car as the distance divided by the <u>time interval as recorded by two synchronized clocks situated at the two markers.</u> The exciting fact is that, so defined, a different value of speed is found. If I, with my watch, find my speed to be exactly 60 miles per hour, the conventionally defined value will be slightly smaller. The difference is, of course, too small to measure. But in situations where relativistic speeds are involved, the difference may be large--in some cases infinite!

Consider the standard twin paradox of Chapter 20. The distance (Earth to star) is 8.7 light-years. The travel-time, as indicated by Traveltwin's watch, is 5 years. Therefore Traveltwin is likely to declare his speed to be:

$$\frac{8.7 \text{ light years}}{5 \text{ years}} = 1.74 \text{ c.}$$

But Earthtwin, whose measurements indicate the trip duration to be 10 years, declares the speed to be:

$$\frac{8.7 \text{ light-years}}{10 \text{ years}} = 0.87c.$$

Good names for these two kinds of speed are <u>proper speed</u> and <u>improper speed</u>, v_{pr} and v_{impr}.

Proper speed is called proper because it is defined as the ratio of proper distance to proper travel-time:

$$v_{pr} = \frac{proper\ distance}{proper\ travel\text{-}time}$$

Importance of Proper Distance: Proper distance is uniquely important for many reasons:
1) It is measured by an observer team that is fixed in the frame with the two markers.
2) The team members can measure the distance as fast or as slowly as they wish.
3) They can measure it again and again.
4) In making their measurements they have no need to use clocks --it is a pure length measurement.
5) The distance value found is unique--greater than found from any other frame.
6) If, fixed in the frame of the markers, there are children's balloons, ball bearings, wheels, and planets, these objects have spherical shape as judged by the given observer team -- whereas teams in all other frames find all of these objects to be elliptical.

Importance of Proper Travel-Time: Proper travel-time is important because it is indicated by the watch accompanying the traveler. Only one watch is involved; no sync is needed; the measurement is a pure time measurement. Also, the value found is unique: smaller than found from any other frame.

In summary, numerator and denominator of the definition of v_{pr} are totally straightforward -- respected from all frames. Eminently proper. Therefore the quotient, v_{pr}, likewise is totally straightforward and proper.

Lesser Status of Improper Speed: The quantity v_{impr} is called improper because the time-interval value used (the denominator) is improper. The value found (by the ground crew) requires use of two clocks and requires sync. Also, the value found is not a minumum or maximum-- not extreme--and is disavowed from all other frames. It well deserves the term improper.

How Are the Two Kinds of Speed Related? Very simply. Proper speed is gamma times the improper speed.

Implication Concerning Light

If, in the twin paradox of Chapter 20, Traveltwin's speed had been much higher, he might have completed the round-trip in a cumulative proper time (t_{cp}) of only one year, or perhaps one month, or one hour.

Now suppose that the "traveler" had been a light pulse. Such a pulse travels much faster than the fastest conceivable person. Therefore if we say that Traveltwin's trip could take only one year of proper time, or one day, one hour......, we are obliged to say: "A light pulse takes no proper time at all."

In other words, the proper speed of light is infinite.

Reconciling the Two Kinds of Speed

Always a team that measures the distance and time arrives at a speed value that is improper.   --Improper because the travel-time value is improper, being not an extreme, not respected from any other frame. Calling this team "the measurer", we arrive at the rule:

"Measurer finds improper speed; never finds a speed exceeding c."

But the traveling object itself does not measure distance. Indeed it cannot measure proper distance. It measures only travel-time-- proper travel-time. And, accepting the proper distance value, the speed value it finds is proper speed. Calling the traveling object "the measuree", we arrive at the rule:

"Measuree finds proper speed; can find speeds exceeding c."

Of What Use is Proper Speed?

One use is in explaining (regarding the twin paradox) how it can be that Traveltwin travels such an enormous distance in such a small time-interval. The explanation is his very high proper speed --speed far in excess of c.

Another use of the concept proper speed is in dealing with a high-speed object's momentum. The momentum is simply mv_{pr}. This is simpler than the expression employing the usual concept of speed--concept requiring inclusion of gamma.

The "proper speed" concept helps explain the standard remark "Nothing can go faster than c." --because if something "went faster than c", its proper speed would have to "exceed infinity."

The concept helps understand why a pulse of light has no frame.

The concept helps understand why a pulse of electrons at, say, 0,999,999,998 c (as was routine at the 6-GeV Cambridge Electron Accelerator) behaves utterly differently from a pulse of light that -- in conventional terms--has almost the identical speed. In terms of proper speed, the speed difference is infinitely great.

Does the Concept "Proper Speed" Suggest that Speed Itself is Frame-Dependent?

Textbooks routinely imply that in any scenario involving a Frame A moving relative to a Frame B, a single value of relative speed applies--"One speed fits all."

But in a typical scenario, one frame is "measurer frame" and the other is "measuree frame". That is, the scenario is asymmetric. Why not accept the idea that the speed situation is asymmetric? The speed of interest to the measurer is "measurer speed" -- improper speed, and the speed of interest to the measuree is proper speed.

Are the two concepts in conflict? No. In the twin paradox, Earthtwin says: "Traveltwin's improper speed is 0.87c." and Traveltwin says: "My proper speed is 1.74 c." No conflict! Earthtwin may add: "His speed is well below c, and so the round-trip took 20 years." Traveltwin may add: "My proper speed was 1.74 c, and so my round-trip took only 10 years."

Could it be that the "proper speed" concept will play an important role in understanding various puzzling phenomena in quantum mechanics, high-energy physics, gravitation?

Possible New Interpretation of C

Could it be that c is a kind of commentary on the space-and-time linkage? Depending on which type of speed you choose to deal with, you do --or do not--encounter c as upper limit. In particular, depending on whether you are measurer or measuree, you do --or do not--encounter c.

Puzzling Breadth of Application of the Improper Speed C

Consider this remarkable fact: the commonly stated speed limit 3×10^8 m/sec, applies -- not just to electromagnetic radiation -- but to baseballs, protons, electrons, gravity, information--everything! It is an overall upper limit on speed in this universe. Therefore it cannot be accounted for by a particular mechanism.

Or consider the great range of wavelengths of the electromagnetic spectrum, from x-rays to radio waves -- a range of 10^{20} or more. Can we believe that x-rays and radio waves have the identical speed, identical to one part in 10^8? Surely no mechanism could account for this!

An easy way out is to say: No mechanisms are involved. No specific number (such as 3×10^8) is fundamentally involved. That number is improper, practical, highly important. But not fundamental.

Infinity is fundamental. For everything, the lowest possible speed (proper speed) is zero -- and the highest possible speed (proper speed) is infinite! What could be simpler?

The highest speed is experimentally demonstrated to be infinite if we define speed as "proper speed". Traveltwin can travel a very large distance in a proper time interval that has no lower limit. Therefore his proper speed has no upper limit. This constitutes experimental proof; it supports relativity theory, not vice versa.

At once we must emphasize that to every practical-minded person, what counts is improper speed--speed as usually defined. Such persons deal with distance values that they have determined, travel-time values they have determined. In nearly all real-life problems, what counts is improper speed. Almost never does anyone have an interest in proper speed.

Yet in seeking better understandings of certain kinds of puzzling phenomena, perhaps the concept proper speed will help clear the way.

Why Does Only One Standard Textbook Discuss Proper Speed?

In perusing 15 textbooks on special relativity, I have found the proper speed concept to be mentioned in only one book: "Introduction to the Theory of Relativity", by F. W. Sears and R. W. Brehme, Addison-Wesley Publ. Co.,1968.

A book by P. W. Bridgman hints at the concept, as does some of George Birkhoff's writings.

Most authors totally ignore the concept.
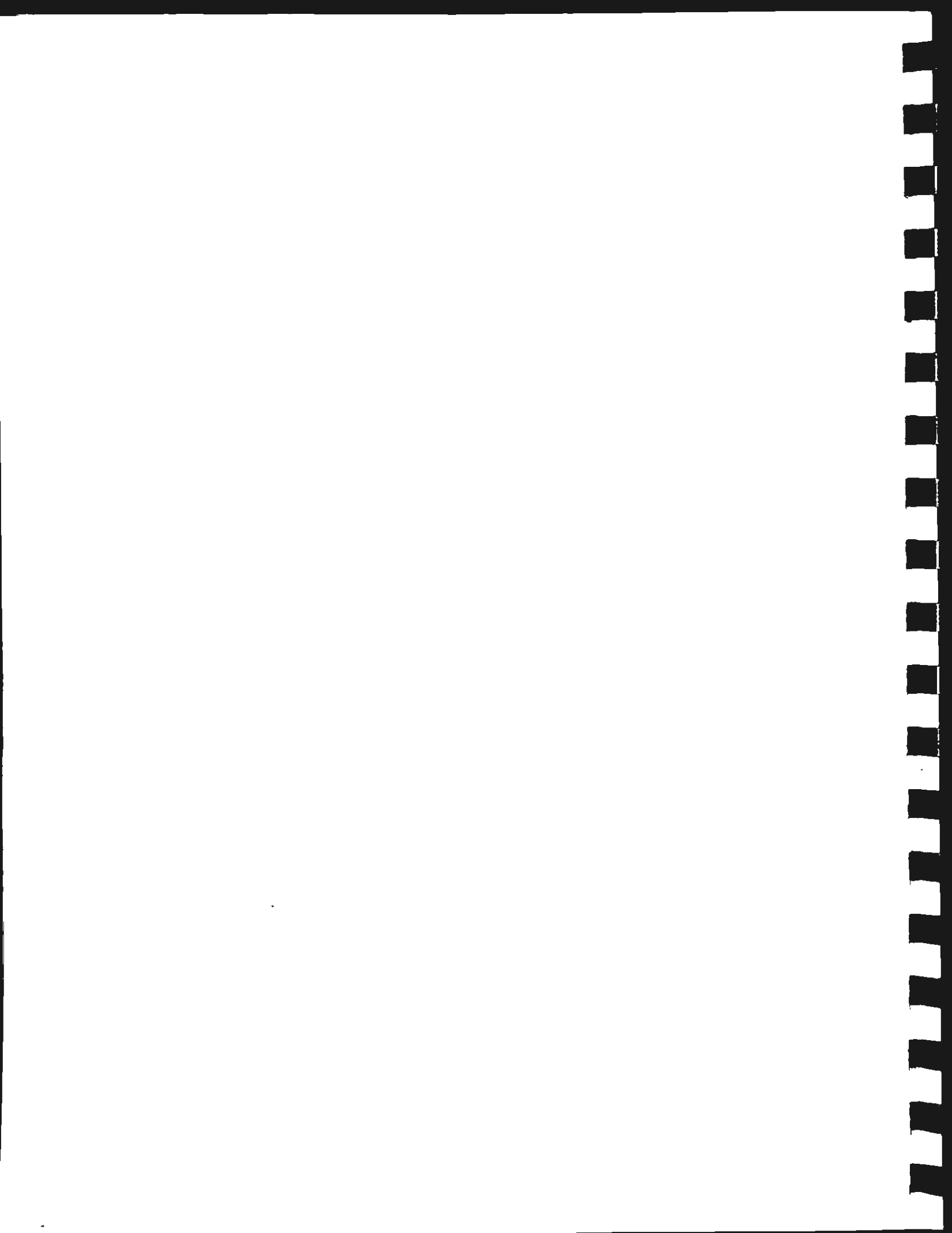
Appendix 1   Do Photons Travel?

It is commonly said that light has wave-like properties and particle-like properties--it is both waves and particles (photons), and it is commonly said that waves and photons travel.

A world expert on light, Prof. Nicholaas Bloembergen of Harvard University, says that waves travel but photons do not. Photon behavior is found during the detectable <u>actions</u> of light: emission, scattering, absorption. But there is <u>no evidence</u> that photons are present during the pure travel phase.

If he is right, then it is incorrect to say: "When a photon is traveling toward......" or "Consider the location of the photon while it is en route to..." or "Consider the frame of the photon..."

Whether he is right or wrong -- whether what travels is waves or photons or (somehow) both --special relativity remains fully valid.

Comment on Reason for the Stability of Photons

If we choose to think of a light pulse as consisting of a bunch of photons --traveling photons, capable of traveling through empty space for billions of years (judged by us)--we arrive at the idea that the proper speed of the photons is infinitely great and the cumulative proper time t_{CP} of the photons is zero (see Chap. 23).

Can we go further and say: "Because their t_{CP} is zero, the photons have no proper time in which to decay or disintegrate; they have no choice but to be stable"?
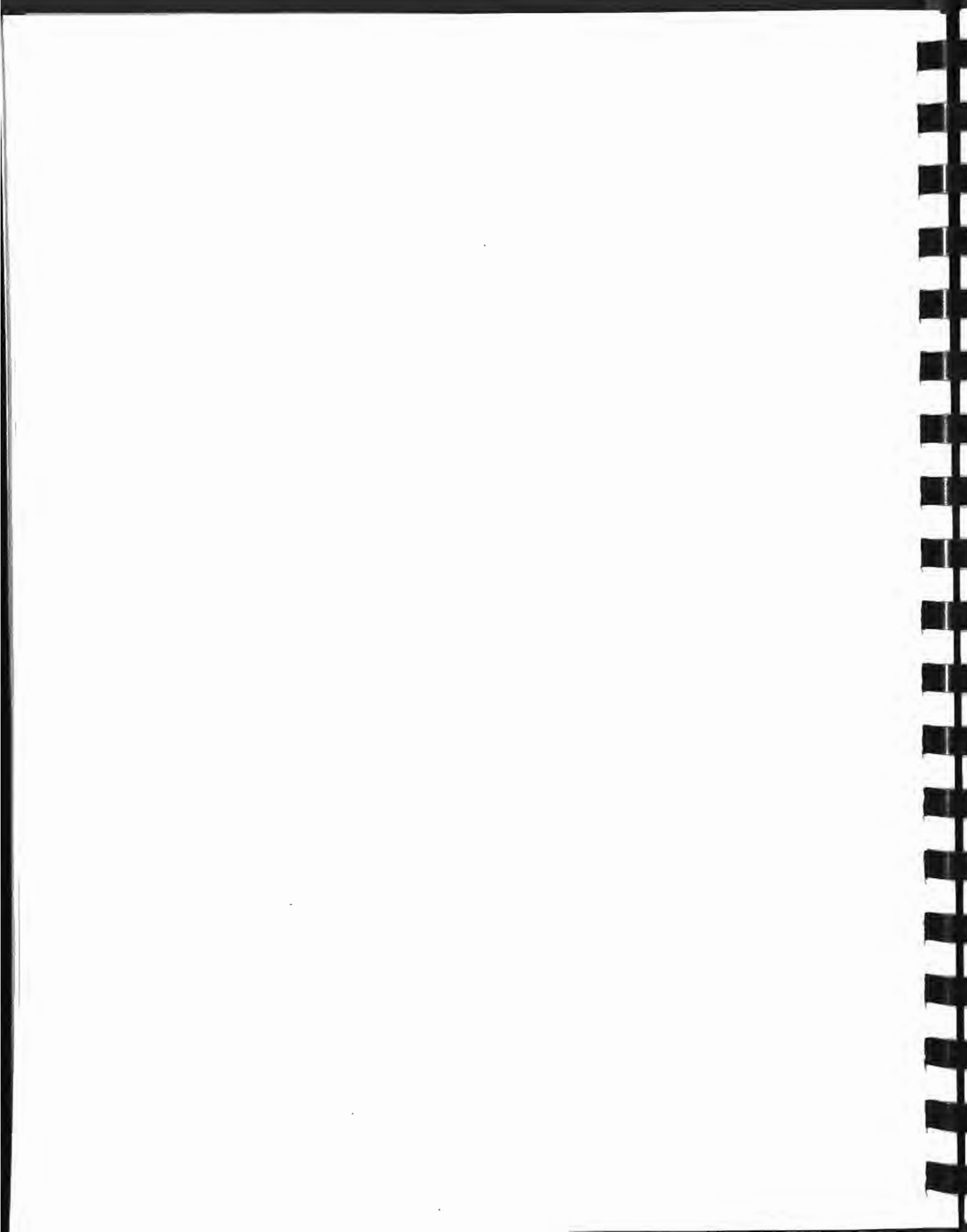
Appendix 2

The Noun "Time" Is Used in Eight Different Ways

1. A number indicating a point on a scale.
    "It is just 3:00 o'clock."
    "What time is the train due?"

2. A number indicating difference between two points on a scale; in other words, an interval, a duration, an increment.
    "What was the best time among the four runners?"
    "It happened in my grandmother's time."

3. A benchmark on a social scale.
    "The time man invented the spear."
    "The time the stone age gave way to the bronze age."
    "The time when spitting was no longer considered polite."

4. Indication of action needed, action regretted, etc.
    "Its high time he stopped smoking."
    "Its time to face the facts."
    "Be patient. He'll do it in good time."

5. Enduring experience.
    "In my middle years I had the best time of my life."
    "Those years in Paris! What a time we had."
    "The prisoners had a very bad time."

6. A commodity.
    "Can't do it! Haven't the time!"
    "Time is of the essence."
    "Time is money. Spend it wisely!"

7. A physical concept.
    "Time and space are linked, according to Einstein."
    "Does time march on, or do we march through time?"
    "Time is an endless stream, with no beginning, no end."

8. Miscellaneous.
    "He came just in time."
    "Time out! I've hurt my foot!"
    "She works part time."

   I regret the fact that most authors of books on special relativity use "time" in three different ways: #1, #2, #7 —with no warning, no apology.
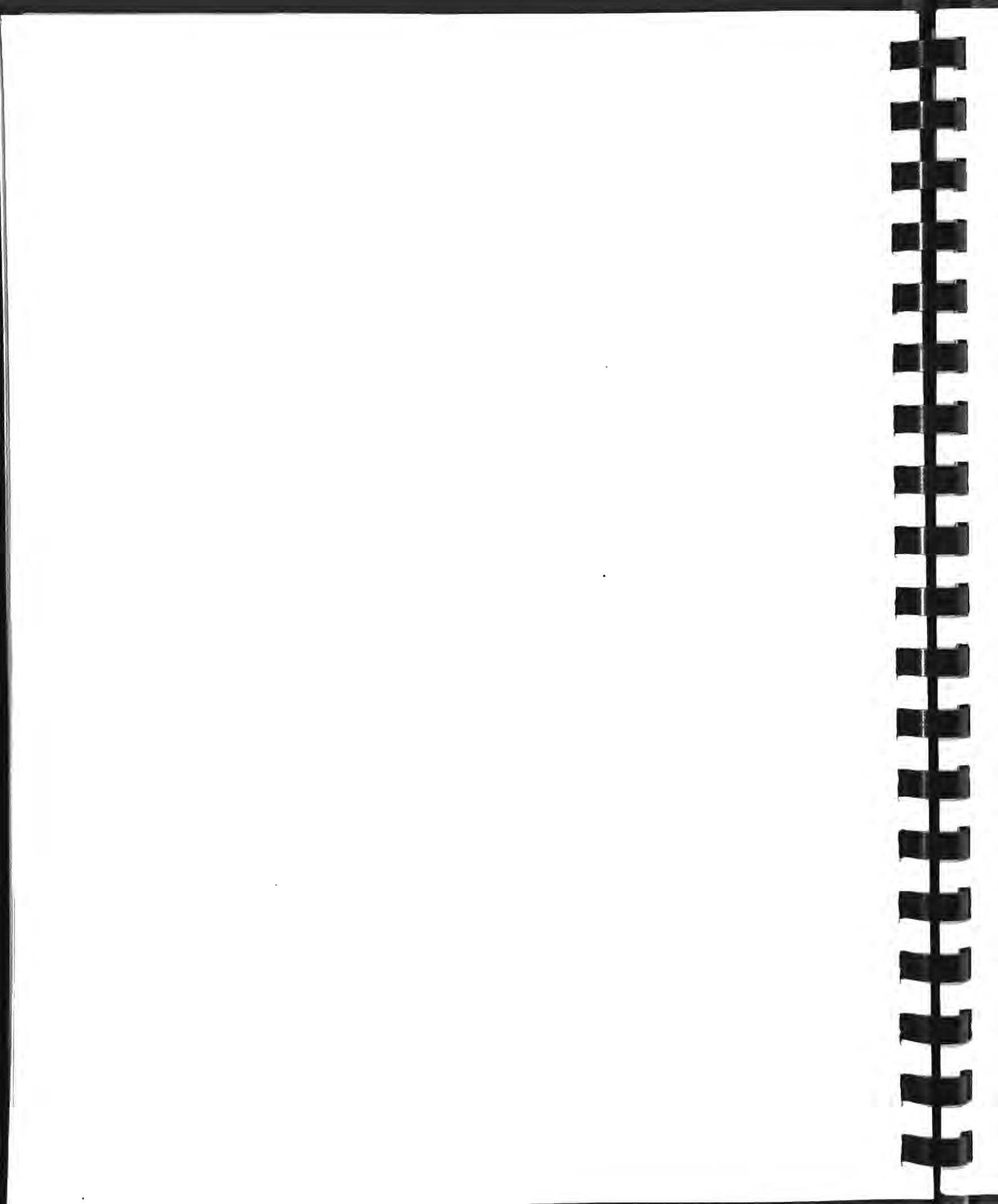
Appendix 3
Some Tricky Questions Concerning
Spacetime Geometry

Which of the following questions are correct?
(Answers are on later page.)

1. Relativistic effects stem from the invariance of the speed of light. Does special relativity apply to a big cave in which there is no light?

2.. So-called "shortening effects" apply not only to tangible objects, such as spaceships, stars, people, atoms, etc., but apply also to empty spaces between objects. Right?

3. Capt. Baker, in his spaceship, holds a rod (aligned along the direction of motion relative to Ableship) parallel to the ship's floor, then throws it toward the floor --in such manner that, as judged by him, it remains parallel to the floor. Can it be that Ablecrew finds the rod to be "short" but still parallel to the floor?

4. Do all observer teams that are at high speed relative to Bakership (which has rest-length 100 ft.) necessarily find, for that ship, length values less than 100 ft.?

5. Captain Able detects two asteroids that are approaching from opposite directions and finds each to be approaching him at speed 0.9 c. Can it be that he finds the distance between the asteroids to be decreasing at the rate 1.8 c?

6. If a clock travels past Capt. Able at high speed, his measurements are suggestive of "clock runs slow"--but only provided that the clock has been accelerated. Right?

7. Two tire-blowouts occurred in different parts of Able's city. Judged by Able, who was stationary in the city, the time interval between blowouts was 50 nanoseconds and the space interval was 50 ft. (Assume light travels 1 ft. in 1 ns.) To Baker, traveling through the city at high speed, the time interval between blowouts was 48 ns. What value of place interval does he find? 52 ft.?

8. Is it true that every pair of events is cotimeable and also is coplaceable? That is, for any given pair of events, can you always specify a frame with respect to which the events were cotimey and specify a frame with respect to which they were coplacey?

9. Is every pair of events distimeable and also displaceable? That is, can you always specify a frame with respect to which the events were distimey—and also a frame with respect to which they were displacey?

10. When a mouse runs diagonally cross the floor of Bakership while the ship is traveling from Earth to Vega, observers fixed in Earthframe find the component of mouse-speed along Bakership's direction of motion to be "slowed." What about the speed-component tranverse to the direction of motion: do they find this also to be "slowed"?

11. Ablecrew, in Earthframe, monitors the light given out by a sodium lamp atop a spaceship that is traveling at high speed relative to Earth. Ablecrew necessarily finds that the light it receives from the sodium lamp has speed c and has an unusually long wavelength, Right?

12. Able finds that as Bakership travels away from him it appears unusually short. Earlier, when Bakership was traveling toward Able, did it appear unusually long?

13. A single pulse of light from a source fixed on Earth spreads out in all directions and is detected by a photocell in Boston at noon and is detected by a photocell in Cambridge a little later. In view of the fact that the two detection events were triggered by a single pulse of light, is the L-interval (Lorentz invariant interval) between the two detection events necessarily zero?

14. Able finds the time interval between two given events to be 60 ns. Does Baker, traveling relative to Able at speed 0.87 c (for which gamma = 2), necessarily find the time interval to differ from 60 ns by the factor 2?

15. The strange results you—on Earth— find for an object traveling at high speed relative to you are strange only provided no massive object, such as a planet or star, intervenes. That is, relativistic effects depend on unimpeded line-of-sight relationships. Right?

16. From different frames, different values are found for the wavelength of light from a lamp that emits monochromatic light in all directions. Right?

Answers to Questions

1. Yes. Special relativity applies to everything in the physical universe, whether or not light is involved. "Invariance of the speed of light" is shorthand for "invariance of the upper limit of speed of light, energy, and matter of all kinds."

2. Yes. Consider an evacuated box that is speeding past you. You find the box to be eifo-shortened, so of course the region of vacuum within the box is likewise eifo-shortened. But remember: the whole notion of "shortening" is "in the eye of the beholder."

3. No. Because Bakercrew finds that the two ends of the rod strike the floor at the same instant--finds the two events to be cotimey-- Ablecrew necessarily finds them to be distimey; the two ends of the rod strike the floor at different times; the rod was "eifo-non-parallel" to the floor.

4. No. A team that is traveling in direction perpendicular to Bakership will find the ship's length to be the rest-length, 100 ft. But teams having any other direction will find eifo-shortening.

5. Yes. The quantity c is the upper limit on speed of matter, light, etc., but does not apply to distance per se.

6. No. The eifo-slowness of a clock depends only on its speed relative to the observer team. The clock's past history is irrelevant. Whether it was accelerated never, once, or dozens of times is irrelevant (assuming, of course, that it was never physically damaged).

7. No. 52 ft. is wrong. He finds 48 ft. Why? Because the L-interval he finds must be the same as that found by Able. Able found: diftor of 50 and 50, which is zero. Baker will find this same result only if the place interval he finds is 48-- to equal his 48 value of time interval.

8. No. If an event pair, judged from some frame, is cotimey, there is no frame with respect to which it is coplacey.

9. Yes, For any given event pair, you can specify a frame with respect to which the events are distimey and also a frame with respect to which they are displacey. (How about two events that occur at the same time and place? Answer: They comprise a single event!)

10. Yes. Earthframe observers find eifo-slowing to occur for all directions of running (of the mouse within Bakership).

11. No. The wavelength may appear unusually long or unusually short depending on whether the lamp is traveling away from the given observer or toward him.

12, No. Eifo-shortening depends just on speed (along the given direction of relative motion), not on the direction (to-vs-fro).

13, No--unless both events were situated along the same "ray" of light.

14. No. The factor gamma applies to the increase in reading of a given clock, or to the length of a given object, but not to the Δt or Δx values between events (unless one of the pertinent frames is a nulling frame).

15. No. The rules of special relativity hold irrespective of intervening objects (provided they are not so massive that general relativity rears its awesome head).

16, No. From no frame is there a definite value. Because a frame has an indefinite extent, some members of its observer-team may be situated "far ahead" and find the light source approaching (and wavelength shortened), other members may be situated "far behind" and find the light source to be retreating (and wavelength lengthened). Furthermore, as the light source passes by a given observer, he finds its wavelength to be changing--decreasing. This whole subject pertains more to Doppler effect than to special relativity.

Appendix 4

Back-of-Envelope, Words-of-One-Syllable Summary of Special Relativity

There is no one true frame (no best frame, no fixed frame) from which to judge time, space, mass, or speed. One frame is as good as the next. All of our great laws are the same in each frame. The <u>speed of light</u>, c, is the same in each frame, and there is no way to change this speed: no change in <u>your</u> speed, or in the speed of the light source, can change the speed at which light comes by you. No star, ship, man, rock, or speck of dust can reach the speed of light. This holds true for all things that can be slowed to a stop: none of these can quite match the speed of light.

If a car zooms by you at high speed, you find that its "length" is less (and its "mass" is more) than when it stands still in your frame. Also, its clock runs "slow". Of course, the deal works both ways: the man in the car finds you to have more "mass" than when you and he are in the same frame.

If two guns (one to your right, one to your left) are fired, and <u>you</u> find they were fired off at the same time, a man who went by at high speed may have found them <u>not</u> to have been fired at the same time. For him, the time span Δt may be large, not nil.

It turns out that time and space are in some sense <u>joined</u>. If two guns, one here and one there, are fired, the time span Δt and the space span Δx are linked in a strange way. Take $\sqrt{c^2(\Delta x)^2 - (\Delta t)^2}$ and note that, though Δt and Δx found from some Frame A are not the same as the Δt and Δx found from some Frame B, yet $\sqrt{c^2(\Delta x)^2 - (\Delta t)^2}$ is the <u>same</u> for each frame -- in fact for all frames.

PF 6/24/70

This summary, prepared by the author in 1971, was put to use by Prof. Edward Purcell of Harvard University in his course on relativity.

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